# *Simulation and Control of Univariate and Multivariate Set-Up Dominant Process*

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# Simulation and Control of Univariate and Multivariate Set-Up Dominant Process

Steven Cox

This thesis explores the use of statistically valid process improvement tools in low-volume applications. Setting out the following research questions: How can the Six Sigma Measure and Analyse phases of a chronic quality problem be statistically validated in a low-volume process? How can a statistically valid approach for process control be implemented in a low-volume process? And how can this tool be extended to fit multivariate processes and can the calculation of control parameter adjustments be automated?

In answer, the thesis presents an enhanced PROcess VAriation Diagnosis Tool (PROVADT) method, driving a Six Sigma improvement project through the Measure and Analyse phases. PROVADT provides a structured sampling plan to perform a Multi-Vari study, Isoplot, Gage R&R and Provisional Process Capability in as few as twenty samples and eighty measurements, making the technique suited to low-volume applications. The enhanced PROVADT method provides a Gage R&R without confounded variation sources, as was the case in the original method, and its practical application was demonstrated through two case studies.

Process control tools for low-volume, high-variety manufacturing applications were developed. An adjustable traffic-light chart, with control limits linked to tolerance and simple decision rules, was used for monitoring univariate processes. This tool, the Set-Up Process Algorithm (SUPA), uses probability theory to provide 98% confidence that the process is operating at a pre-specified minimum level of $C_p$ in as few as five samples. SUPA was extended to deal with high-complexity applications, resulting in multivariate SUPA (mSUPA). mSUPA maintains SUPA's principles, but presents the information about multiple process features on one chart, rather than multiple univariate charts. To supplement the mSUPA tool, a theoretical method for calculating optimal process adjustment when a multivariate process is off-target was introduced, combining discrete-event simulation and numerical optimisation to calculate adjustments.

Doctoral Thesis

# Simulation and Control of Univariate and Multivariate Set-Up Dominant Process


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*A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy*

*in the*


Mechanics Group

School of Engineering and Computer Sciences


2015

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# Abbreviations

**ACC** Acceptance Control Chart

**AMT** Advanced Manufacturing Technology

**ANOVA** ANalysis Of VAriance

**CAD** Computer-Aided Design

**CNC** Computer Numerical Control

**CtQ** Critical-to-Quality

**DMAIC** Define-Measure-Analyse-Improve-Control

**DoE** Design of Experiments

**EPC** Engineering Process Control

$\bar{X}\&R$ **Chart** Mean and Range Chart

**MSA** Measurement System Analysis

**mSUPA** Multivariate Set-Up Process Algorithm

**PROVADT** PROcess VAriation Diagnosis Tool

**PIMS** Process Improvement Methods Simulation

**R&R** Repeatability and Reproducibility

**RtR** Run-to-Run

**RQ** Research Question

**SB**$\bar{X}R$ Small-Batch $\bar{X}R$ Chart

**SPC** Statistical Process Control

**SUPA** Set-Up Process Algorithm

**TQM** Total Quality Management

# Nomenclature

$\alpha$ Number of time periods

$\alpha_q$ Risk of false fail when qualifying a process

$\beta$ Number of consecutive samples

$\delta$ Difference between process mean and process target

$\varepsilon$ Total number of appraisers taking measurements in a location

$\mu$ Process mean

$\Phi$ Normal distribution function

$\varphi$ number of total measurements

$\sigma$ Process standard deviation

$\sigma^2$ Multivariate process covariance

$\sigma_m$ Standard deviation of repeated measurements

$\sigma_A^2$ Reproducibility variation

$\sigma_E^2$ Repeatability variation

$\sigma_P^2$ Part-to-part variation

$\sigma_{RR}^2$ Measurement repeatability and reproducibility variation

$\sigma_T^2$ Total variation

$C_g$ Repeatability metric

$C_{gk}$ Repeatability and bias metric

$C_p$ Process capability precision metric

$C_{pk}$ Process capability precision and accuracy metric

$G_L$ Position of lower green zone limit

$G_U$ Position of upper green zone limit

$H^2$ Maximum Mahalanobis distance

$k$ Number of consecutive green units needed to validate a process

$L$ Lower tolerance limit

$LCL$ Lower control limit

$n$ Sample size

$n_m$ Number of standard deviations

$m$ Number of subgroups

$P$ Percentage of the CtQ tolerance

$P(g)$ Probability of a a CtQ falling in a green zone

$P(q)$ Probability of qualifying a process

$P(y)$ Probability of a a CtQ falling in a yellow zone

$P/T$ Precision-to-tolerance ratio

$R$ Subgroup range

$\bar{R}$ Mean of the subgroup ranges

$R_E$ Average range between repeat measurements

$r_i$ Repeated measurements per appraiser

$R_P$ Average range between repeat measurements at different locations

$r_{total}$ Total measurements per sample

$S$ mSUPA minimum process covariance

$T$ CtQ target

$t$ Number of consecutive yellow units needed to identify a nonconforming process

$U$ Upper tolerance limit

$UCL$ Upper control limit

$X$ Measured CtQ value

$\bar{X}$ Estimated subgroup mean

$\bar{\bar{X}}$ Mean of the subgroup means

$\bar{X}_m$ Mean of all measurements

$x$ Measured CtQ vector

$X_{PC}$ Percent tolerance Pre-Control value of measured CtQ

$X_{ref}$ Reference value

$A_2$ Statistical constant

$d_2^*$ Hartleys statistical constant

$D_3$ Statistical constant

$D_4$ Statistical constant

# Statement of Copyright

# Acknowledgements

I start by saying a big thank you to my supervision team: Dr. Apostolos Kotsialos and John Garside. Apostolos' technical knowledge was a crucial bouncing board for developing my simulations and his scrutiny of my papers led me to learn to some English grammar. John's shear enthusiasm for industry and process improvement inspired my research path from first meeting him as an undergraduate; my thoughts may have stagnated without the almost daily conversations about manufacturing, industry, quality and Rugby League! I must mention and thank Prof. Val Vitanov, who supervised my MSc and whose untimely passing before this body of research began saddened all who new him; however, it was his inspired idea to use simulation to test process improvement ideas for low-volume manufacturing that underpins this thesis. Also, I am grateful to the School of Engineering and Computing Sciences for providing the studentship that allowed me to conduct this research without the burden of tuition fees.

This research would also not have been conducted without the support of industry; as such, I thank all the companies who allowed me to visit and observe their processes and practices. In particular, thank you to Grant Lennon, at Rettig, and Chris Gill, at Coveris, for supplying data and allowing its publication. I am also indebted to Durham University undergraduates: Virginnia Elton, at Rettig, and João Victor Marmo and Lorena Cunha, at Coveris. Their student projects in industry collected the data necessary to apply the PROVADT method developed in this thesis.

I'm deeply grateful for my friends and family, whom there are to many to list, but all played a part in getting me through this endeavor. Especially, I must thank my long suffering partner Lucy, whose help and support has been ever present.

# Chapter 1

# Introduction



## 1.1 Scope

Over the past two decades, the UK manufacturing sector has been moving from a high-volume to a low-volume production paradigm. In this low-volume paradigm, products are made by Advanced Manufacturing Technology (AMT), such as multi-axis machine tools, laser-cutting, micro-manufacturing and 3-D printing. The set-ups of these processes are being regularly changed to manufacture a high-variety of high-value products that are typically produced in small numbers, with high-precision and complexity.

The low-volume nature of these processes and the high-complexity of products, generates common issues. For example, the difficulty of applying data driven analysis to solve chronic quality problems, or Statistical Process Control (SPC) to identify process changes. This problem is confounded with the extra variation created by regular set-up changes to the process. Research in this field has shown, that to deal with quality problems in these processes, engineers are using subjective reasoning rather than objective analysis [1]. This lack of objective analysis is highly significant, because unnecessary changes to processes made by operators can increase the variation in final products. On the other hand, failure to perform corrective action to a problem process can result in final products with Critical-to-Quality (CtQ) parameters outside of their design tolerances. These types of actions are more likely to occur when decisions are made based on a subjective approach instead of objective analysis of process data.

In order to address these problems, this thesis explores the current state-of-the-art process improvement methodologies and techniques, with a specific reference to low-volume manufacturing. New techniques are then developed and tested, using a novel simulation approach, to objectively improve and optimise AMT in a low-volume production mode. The next sections establish the wider context for the implementation of these new techniques and outline the research questions addressed throughout the thesis.

## 1.2 Context

### 1.2.1 Advanced Manufacturing Technology (AMT)

An AMT is defined as *'an automated production system of people, machines, and tools for the planning and control of the production process'* [2]. In the current manufacturing environment, there is an increase in companies utilizing AMT to cope with shorter product life-cycles and increasing product variety and consumer choice [2, 3]. The UK government sees AMT as a key area for future economic growth [4–6]. This view is supported by a Royal Bank of Scotland report [7], which identifies high-value engineering and its utilization of AMT, as a source of growth in the UK manufacturing sector. Given the economic importance of AMT, it is important to identify key factors and to research solutions to manage their affects on product quality. These factors include the low-volumes of production and the high-complexity and precision requirements of products. An additional aspect of AMT processes, which does not directly effect quality but raises the stakes of getting product quality right, is the drive towards high-value products. The remainder of this section discusses these product quality affecting factors.

### 1.2.2 Low-Volume and High-Variety

Two complimentary trends in AMT, are processes with low-volume production runs and the *'changeability of manufacturing'* [8]. The low-volume nature means that a single product-type is made in batches as small as one unit. To make sure that AMTs do not stand idle they, therefore, must produce multiple product variants in a single process. This makes them high-variety and changeable processes, which reduces the need for expensive bespoke equipment to make each product [9]. However, this production flexibility has proved problematical for operators seeking to gain increased performance of these high-variety processes. Operators struggle to perform classical statistical tests, especially in the absence of data necessary to apply objective analysis. This applies to both the identification of root causes of process variation and ongoing process monitoring to prevent defective products. The difficulty of applying classical statistical tests has led to the abandonment of objective analysis in favour of a subjective approach [1], utilizing engineers' opinion over data driven improvement. It is a situation that is exacerbated by the variation induced by regular set-up changes, known as set-up dominant variation [10].

Research into this field is imperative to ensure objective analysis and improvement of these processes. In particular, advancing the development of analysis tools that focus on the diagnosis of the root cause of variation and the monitoring of ongoing processes is important. Developing these techniques is required to achieve statistical rigour under the conditions of low-volume manufacture and are even more critical under the requirements of high-precision.

### 1.2.3 High-Precision

*'Manufacture with higher precision'* is a second trend in AMT [11]. An example of this requirement is observed in the aerospace industry, with the manufacture of precision aerofoils for jet-engines. Making aerofoils with CtQs that are closer to their design targets, can contribute to a final engine with small in-service efficiency saving. However, even small improvements in efficiency, in the order of 0.1%, saves the aircraft operator tens of millions of pounds in fuel costs over the engines service life. In addition to fuel savings there are reductions in green-house emissions, which makes it possible for aircraft operators to comply with ever tighter emissions regulations. This results in products that are made with ever smaller tolerances on CtQ design features; in order to achieve the improved in-service performance and customer satisfaction. However, these tighter tolerances place additional pressure on manufacturing operations to produce products with-in design tolerances that are at the limits of their manufacturing capabilities. This issue is exacerbated given the high-value of these products, as companies cannot afford to produce defective products.

### 1.2.4 High-Value and the 'Cost of Quality'

The increasingly high-value nature of products manufactured in low-volume paradigms, places greater importance on the cost-of-quality, or more accurately the cost-of-poor-quality. As discussed in the previous section, placing tighter tolerances on on CtQ design features, makes producing a product outside its tolerances easier. The products that fall outside design tolerances are either reworked or scrapped, in both cases adding financial expense and strain to the capacity of the manufacturing process. This rational in which products that are within tolerance are *'good'* and products that are outside tolerance are *'bad'* is known as *'worst-case tolerancing'* [12].

A model of how worst-case tolerancing effects the cost-of-quality is shown in Figure 1.1, where, the lower tolerance limit, $L$, the upper tolerance limit, $U$, process target, $T$, and cost-to-quality represents the financial loss to the manufacturer as a result of an off-target CtQ. In this model only products that are outside their tolerances are considered to add to the the financial cost-of-quality and it is a view that is commonly held on the shop-floor. This leads to operators steering their processes within tolerance but not necessarily on-target.

In practice, the cost-of-quality effect is much more subtle. Consider the example of an aerofoil for a jet engine. If two aerofoils are selected, one that has all its CtQ features near their design targets and the other with all of its CtQ features just inside there upper tolerance limit, both are accepted for service, but do they both have the same in-service performance?

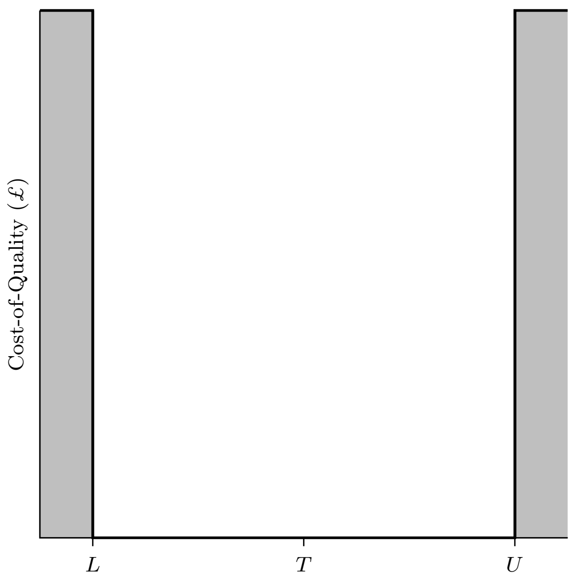


Figure 1.1: Effect of worst-case tolerancing model on the cost-of-quality.

As described in the previous section, the closer the aerofoils are to their design targets the better their performance is, which leads to significant fuel cost savings over the life of the engine. Therefore, the actual relationship between a products CtQ feature and the cost-of-quality is better described by Taguchi's quality-loss function [13]. Figure 1.2 shows the quadratic function of Taguchi's quality-loss relationship. The further away a CtQ is from $T$, the greater the cost-to-quality, as fewer products function optimally and, ultimately, fail.

It is, therefore, critical that products from manufacturing processes are made with precision, i.e. as close to the design target as economically possible. To achieve this, operators need to be supported with appropriate tools to steer their processes not only within tolerance, but also on-target. A process whose output CtQ is on-target and has a small amount of variation relative to its design tolerance is said to have a high capability and maintaining a high capability minimises the cost-of-quality effect. In addition, maintaining a globally high capability on a complex product which may have in excess of 20 CtQ features greatly increases the challenge for manufacturers.

### 1.2.5 High-Complexity

The increasing complexity of products is a fourth trend in AMT. Products with multiple CtQs are manufactured by fewer and fewer processes. Using the example from the aerospace industry of the production of aerofoil parts for jet engines, improvements to engine performance have been gained by modifying the shape and geometry of the aerofoils. This has lead to aerofoils that are becoming ever more complex with an increasing number of CtQs. Traditionally, these parts would have undergone several separate machining processes such as turning, milling and grinding, but with the development of AMTs, these are now produced by single machining centres. This has given rise to situations where products with in

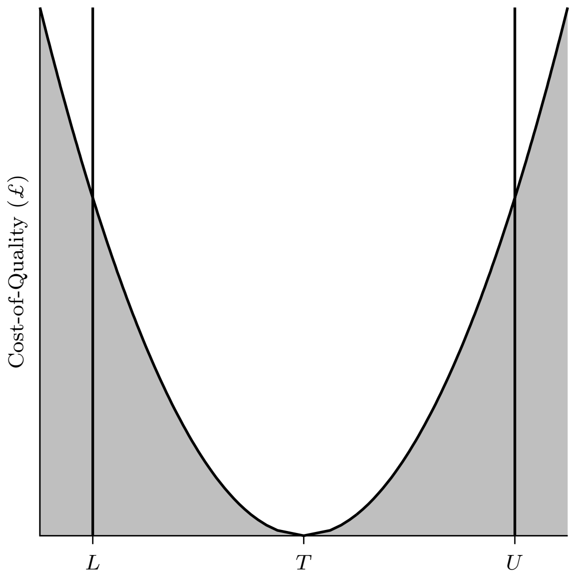


Figure 1.2: Effect of Taguchi's quality-loss function on the cost-of-quality.

excess of 20 CtQ features can be manufactured by a machine tool with a 5-axis, or control parameters, to which an operator can apply offsets to improve the final products compliance with its design targets. As has been shown in the preceding sections, it is important that the global precision of the part is maintained and, therefore, all CtQs are as close to the design target as possible. However, when there are significantly less control parameters on the process than CtQ features on the product, applying an offset to a single control parameter can effect multiple product features. This means a single control parameter is correlated to multiple CtQ design features, and the relationship between the process control parameters and product design features is defined mathematically by a set of unsolvable linear equations. Currently, optimising these processes, and the defining set of unsolvable linear equations, is left to the process operator; who rely on intuition and experience. Therefore, operators need to be supported to steer their processes not only on-target on a individual CtQ basis, but also on-target on a global product level. Also, given the complexity of the low-volume manufacturing paradigm, when process improvement tools are developed to support operator's actions, these tools should present information that is understandable and actionable by the operator.

### 1.2.6 End-User Requirements

There is a trend for the *'democratization of statistics'* in industry [14]. The thrust of this research, supports this trend by reviewing and developing data driven analysis and control tools to support operator's of low-volume manufacturing processes. However, the potential end-users must be considered at all-times to avoid this research falling into the *'serious disconnect between academic research in statistics and quality control and actual practice'* [14]. These end-users, could range from experienced quality engineers with a high level of statistics training, to process operators who have no background in statistics. Therefore, methods that are developed are underpinned by sound statistical theory, but have an interface appropriate for the end-user. If the application of the methods developed involves complicated and convoluted stages, no matter how powerful they are statistically, the chances of them being used is minimal. Hence, this research follows the observation, accredited to the statistician Churchill Eisenhart [15], of:

*...the practical power (of a procedure) is a product of the Mathematical power by the probability the procedure will be used...*

## 1.3 Research Questions

As described in Sections 1.1 and 1.2, this research reviews and develops, tools and techniques to support operators identify root causes of variation in and maintain control of low-volume manufacturing processes. Given the context provided in Section 1.2, the following Research Questions (RQs) have been devised for the remainder of this thesis to answer:

**RQ1** *When improving low-volume processes, what is the relevant process improvement framework to structure the deployment of statistical tools?*

**RQ2** *Within a proposed framework, how can the measurement and analysis of a chronic quality problem be statistically validated in a low-volume process?*

**RQ3** *What is required to provide a practical and statistically valid approach for process control in a low-volume process?*

**RQ4** *How can a low-volume process control tool be extended to fit multivariate processes?*

**RQ5** *When a low-volume multivariate process control tool signals the process is off-target, how can the calculation of control parameter adjustments be automated?*

## 1.4 Thesis Outline

A literature review to determine the state-of-the-art in quality improvement in low-volume manufacturing is provided in Chapter 2. Research conducted to develop a technique known as PROcess VAriation Diagnosis Tool (PROVADT), which aids the identification of root causes of low process capability in low-volume manufacture, is detailed in Chapter 4. Research conducted to develop a technique known as Set-Up Process Algorithm (SUPA), which monitors an assumed capable process and signals to a user when it is off-target or exhibits excessive variation is presented in Chapter 5. Chapter 6 develops the SUPA technique further to monitor a multivariate process, to determine if it is off-target. Further to this, work presented in Chapter 7 details potential methods of automating the calculation of corrective action needed when a multivariate process is off-target.

# Chapter 2

# Literature Review I - Process Improvement Methodologies



## 2.1 Introduction

### 2.1.1 Outline

Industry has developed process improvement methodologies to assist operators improve manufacturing process performance and product quality. This is achieved by promoting a structured and consistent problem solving strategy. In this context, a process improvement methodology is defined by De Mast [16] as: *'a coherent series of concepts, steps (phases), methodological rules and tools, that guide a quality professional in bringing the quality of a process or product to unprecedented levels'*. However, the predominant application of process improvement methodologies in manufacturing has focused on high-volume/mass-production implementations. This chapter reviews both academic and industrial literature on process improvement methodologies, with the purpose of identifying the most appropriate framework to assist operators working in a low-volume paradigm. The identification of a suitable process improvement methodology, informs the remainder of the research into specific process improvement tools for low-volume applications.

In broad terms there are multiple examples of process improvement methodologies, including: Lean, Total Quality Management (TQM), Six Sigma, Lean Six Sigma, Taguchi Methods and the Shainin System. To start this literature review, it is important to identify which of these process improvement methodologies fit the definition provided by De Mast [16].

### 2.1.2 Context

#### 2.1.2.1 Lean Manufacturing

Lean manufacturing is a quality philosophy that aims to eliminate waste, or non-value added activities, from a process [17–21]. These non-value added activities are divided into seven forms of waste and targeted by applying specific Lean tools [17]. However, Lean tools are process-flow orientated and the philosophy does not have an underpinning problem solving structure, as specified in the De Mast [16] definition.

#### 2.1.2.2 Total Quality Management (TQM)

In the same regard TQM offers a quality philosophy. To achieve process improvement TQM applies statistical techniques, but they are not applied with a systematic step-wise approach [16, 22, 23]. Lean and TQM are not not reviewed further in this chapter, as they do not meet the requirements of the definition of a process improvement methodology. Therefore, they are not a suitable solution.

#### 2.1.2.3 Six Sigma

Six Sigma is a methodology that aims to reduce variation in a process by following a structured problem solving algorithm [24–26]. This structured problem solving algorithm, known as Define-Measure-Analyse-Improve-Control (DMAIC), is accompanied by a set of statistical tools to achieve data driven process improvement. There is also literature which supports the current use of Six Sigma in UK industry for low-volume processes [1].

#### 2.1.2.4 Lean Six Sigma

Lean Six Sigma is the integration of Lean tools to eliminate waste and Six Sigma methodology for variation reduction [17, 27, 28]. Both Pepper and Spedding [17] and Bhuiyan and Baghel [28] identify that research into Lean Six Sigma has not matured, with a common definition yet to be defined. When Lean Six Sigma has been used in a problem solving context the DMAIC structure has been used for process investigations.

The key component of both Six Sigma and Lean Six Sigma, in the context of this thesis, is the DMAIC problem-solving strategy. This strategy will be further reviewed in this chapter.

#### 2.1.2.5 Taguchi Methods

Taguchi methods, like Six Sigma, focus on reducing variation utilizing statistically derived experiments [16]. As an approach, Taguchi methods are more suited to reducing process variation by improving system robustness through optimization experiments [13, 29, 30].

This approach is not considered further as a process improvement methodology, as it does not provide a framework, but rather focuses on experimentation.

#### 2.1.2.6 The Shainin System

The Shainin System is a methodology that reduces process variation by employing a systematic approach to problem solving [31, 32]. This approach uses a novel *'narrow-down'* strategy, whereby, potential causes of variation are eliminated until the root cause is found. This narrow-down strategy, is rigourously applied by the overall Shainin System algorithm. The algorithm provides users with a step-by-step approach to problem-solving. Therefore, the Shainin System is examined more thoroughly in the remainder of the chapter.

### 2.1.3 Scope

The scope of this literature review includes published works in English and predominantly post-1980, when there was a boom in the industrial development and application of process improvement methodologies. There is increasing interest and literature on using process improvement methodologies to solve service sector and overall business performance issues, however, this material was not reviewed and is outside the scope of this research. Individual process improvement tools that supplement a process improvement project are not explored in this chapter, as these are addressed in chapter 3.

The objective of this literature review is to investigate process improvement methodologies specifically as problem solving strategies for low-volume manufacturing processes. The remainder of this chapter focuses on two specific approaches: Six Sigma and the Shainin System. A wealth of material on Six Sigma was available, including textbooks, technical papers, literature reviews and case studies. In comparison, the literature on the Shainin System was both sparse and controversial, which is a result of its proprietary development [33].

## 2.2 Six Sigma

### 2.2.1 History

Many manufacturing companies are implementing the Six Sigma process improvement methodology, to improve bottom-line financial results by focusing on product quality and reducing process variation [24, 26, 28, 34–36]. The methodology was originally conceived at Motorola during the mid- to late-1980s [24, 26, 28, 34, 35, 37, 38]. Motorola was subsequently the first company to win the Baldrige Award in 1988 as a direct result of its Six Sigma initiative [26, 34, 36, 38]. The Baldridge award is a US presidential award for performance excellence. This lead to Six Sigma being adopted at major American manufacturers such as Allied Signal and General Electric [34–36, 38], with new implementation successes reported ever since, see [35, 39–41].

There is no one clear or concise definition of Six Sigma [25, 38]. Aboelmaged [34] identified 8 different definitions in a comprehensive review based on an analysis of 417 articles published between 1992 and 2008. Despite its frequent occurrence in both industry and academic literature, Aboelmaged [34] found that 64.3% of Six Sigma literature focuses on its original application as a manufacturing process improvement methodology; whereas, the remaining papers explore its uses as a business, service and transactional process improvement methodology. Recent work on Six Sigma as a manufacturing process improvement methodology tends to focus on the integration of Lean thinking and tools into a holistic process improvement initiative [17, 20, 28, 34, 42, 43]. In addition to this, Six Sigma implementations are divided into an inner-DMAIC loop, describing individual problem solving projects, and an outer-DMAIC loop, detailing the strategic implementation [44]. The application of Six Sigma is continually broadening [28, 34] and it is the author's belief, that the lack of clarity in a definition stems from this.

In this research, Six Sigma is examined as a purely manufacturing orientated problem solving strategy and in this context common defining features are found. This includes Six Sigma [35, 38, 42, 45]:

- as a process improvement methodology;
- employing a structured approach to problem solving DMAIC;
- focusing on variation reduction;
- having an objective of reducing the number of defects to as low as 3.4 parts per million opportunities;
- achieving bottom-line financial results.

To assess the potential of Six Sigma in a low-volume paradigm, its structure needs to be detailed and its strengths and weaknesses examined. This structure is outlined in the following section.

### 2.2.2 Structure

Six Sigma utilizes the five-phase DMAIC methodology to link the application of process improvement tools in a sequential order [25, 27, 34, 45]. Figure 2.1 shows how the DMAIC process improvement cycle is organised. When an improvement project is in the Define phase, the scope, goals and financial targets are identified [27]. In the Measure phase, baseline measures of process capability and measurement system capability are assessed [27]. During the Analyse phase, the root causes of variation, that effect the output variables linked to the project goals, are identified [27]. In the Improve phase, pilot trials of potential solutions are run before executing a full-scale implementation [27]. For the Control phase, process

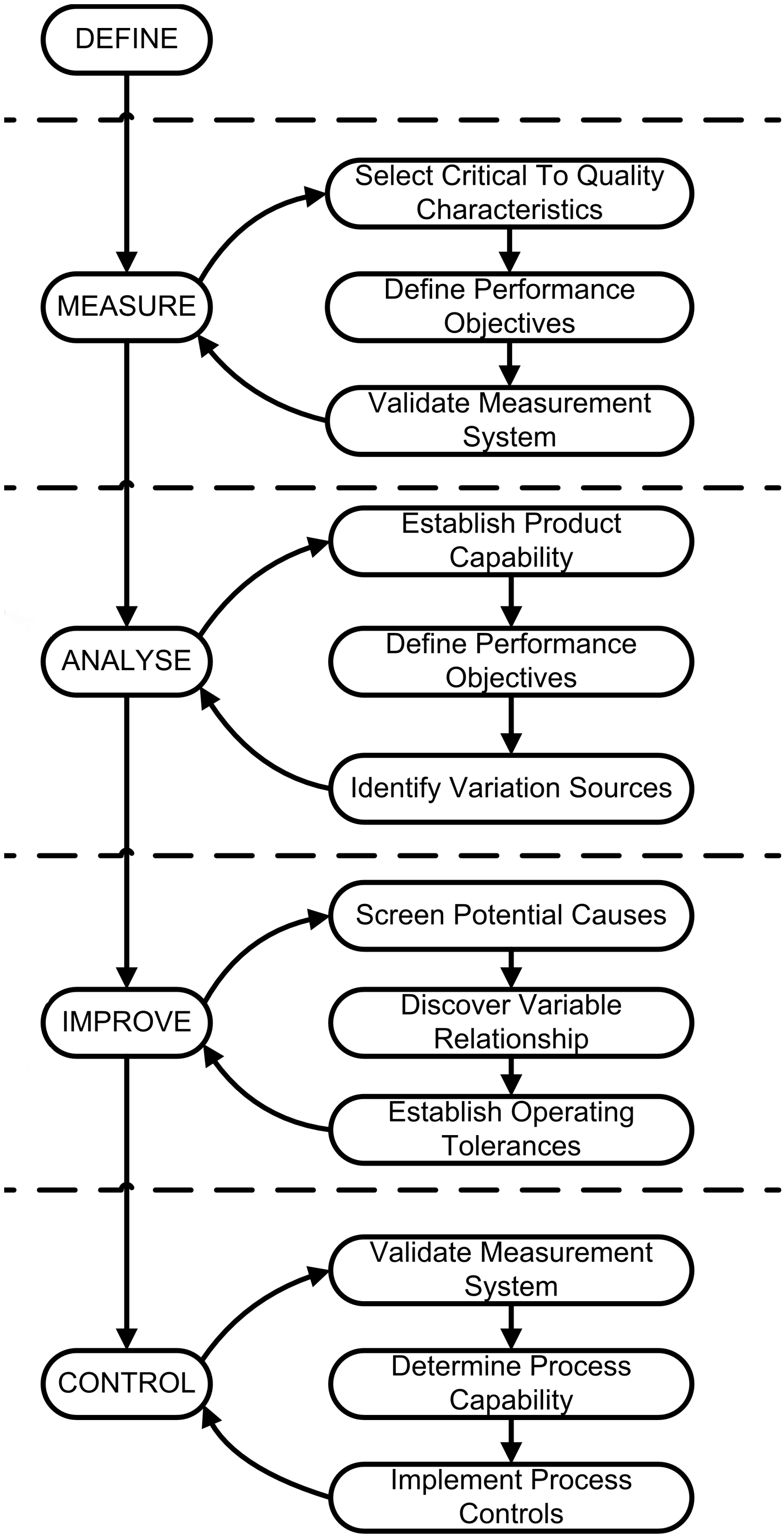


Figure 2.1: Reproduction of Six Sigma's DMAIC quality improvement cycle [37].

procedures are developed and applied to maintain the improvement gains achieved in the previous phases [27].

To support each phase, of the DMAIC methodology, a toolkit of statistical and non-statistical techniques is outlined in Six Sigma literature [25, 27]. Specific tools, from the toolkit of statistical and non-statistical techniques, are used at each stage of the framework to allow data driven decisions to be made about the process. An overview of the classic techniques used at each stage of DMAIC can be found in Pande et al. [24] and George et al. [27]. However, a review of the specific techniques and tools, relevant to this thesis, is made in chapter 3.

### 2.2.3 Successes

The prevalence in industry of Six Sigma as a process improvement methodology is largely due to its many high profile success stories. This success started with originators Motorola, reporting savings of \$16 billion from 1986 to 2001 and followed by other early adopters including GE saving \$4.4 billion from 1996 to 1999 and Honeywell saving \$1.8 billion from 1998 to 2000 [46]. Alongside these impressive financial results are equally impressive technical results. For example, LG Electronics' appliance business achieved a 50% reduction in defects between its start in 1995 and 2000 [47].

Further to these organisation level results, there are also many individual case study successes reported. Suresh et al. [41] reported a 40% reduction in defects in a automobile piston ring manufacturing process. Gijo et al. [40] reported a \$80,000 saving associated with improvements to scrap and rework rates in a high precision grinding process for a diesel engine injector part. For a similar grinding process, Gijo et al. [39] reported a reduction in defect rates from 16.6% to 1.19% leading to a financial saving of \$2.4 million per annum.

The result of these successes, both financial and technical, saw 35% of US organisations having a Six Sigma program in place by 2006 and 82 of the 100 largest US organisations having Six Sigma by 2007 [46]. Despite these successes Six Sigma is not without its problems or critics. These issues will be explored in the following section.

### 2.2.4 Issues

In traditional high-volume Six Sigma applications, there is a weakness in the Analyse phase of the DMAIC methodology. During this phase, Six Sigma uses subjective, non-statistical approaches, for example brainstorming, Ishikawa diagrams and cause-and-effect matrices, to form a causal hypothesis about the root cause of a quality problem [24]. If these types of subjective techniques are used to identify potential root causes, it is possible that the real root causes of a quality problem are missed. The hypothesis formed, is then validated using complex statistical analyses, for example full or fractional factorial Design of Experiments (DoE). This approach can also be unproductive if not carefully applied. A fractional factorial is a screening technique [48], which reduces the numbers of experiments that are used, making an investigation manageable but at the expense of understanding higher order interaction effects [10]. If full factorial designed is used, based around a small number of root causes identified using subjective analysis, it is possible that the real root causes of a quality problem are missed. On the other hand, if the DoE is based around a large number of potential root causes, the experimentation becomes impractical. For example, applying a two-level Full Factorial design when there are five or more factors is cumbersome (with five factors a minimum of $2^5$ or 32 experiments is needed). This weakness in Six Sigma's *'exploration'* of a quality problem is identified by De Mast et al. [49], who state:

> *...Six Sigma seems the most complete strategy...[but] the guidance and tools that are given for the exploration phase lack clear structure and coherence...*

This becomes particularly important to overcome when a low volume of product is available to test. Hence, as UK manufacturing has moved towards the use of AMT, to provide low-volume, high-complexity and precision processes, so has the application of Six Sigma [1]. However, applying classic statistical tests, associated with Six Sigma, to low-volume processes is difficult due to the lack of available data and opportunity to trial forced experiments. Therefore, practitioners place heavy reliance on non-statistical techniques to drive improvement projects. This reliance on subjectivity in the Analyse phase is questionable and deviates from Six Sigma's underpinning mantra of *'data driven improvement'*.

## 2.3 The Shainin System

### 2.3.1 History

The Shainin System, also referred to as Statistical Engineering, originates in the 1940's when Dorian Shainin, whilst working with Joesph Juran and Leonard Seder, identified that the Pareto principle[1] applied to the causes of variation in manufacturing processes [31]. This overtime led to the development of statistically derived tools to identify the causes of variation, specifically addressed in chapter 3. The Pareto principle also led to the Shainin System, as a problem-solving system that applies these tools in a systematic procedure to achieve quality improvement [32].

The underlying logic of the Shainin System as a problem-solving methodology was reported as early as the 1950's by Shainin [50]. However, it's well defined algorithmic structure was not reported until the 1980's, see [51]. It is this well defined approach that is discussed in this chapter and the structure is outlined in the following section.

[1]The Pareto principle is the concept that approximately 80% of effects are a result of 20% of potential causes [10].

### 2.3.2 Structure

The Shainin System aims to find the major, secondary and tertiary causes of variation. These inputs are known as the Red X, Pink X and Pale Pink X, respectively which affect the output, known as the Green Y. To determine the Red X, convergence techniques are implemented within the Shainin System algorithm, a method known as *'narrow-down'*. The process of converging on a Red X is known as a Y to X approach, where differences in results (Ys) are analysed to rule out unimportant factors, narrowing down to the Red X [31].

The Shainin System approach is under-pinned by the algorithm [32, 51] shown in Figure 2.2. The algorithm initially considers all potential causes of variation at the *'Generate Clues'* stage. At this stage off-line experiments are performed to eliminate variables in a process that do not have an effect on the overall variation, without disrupting the process settings. This allows fewer experiments to be performed with the identified suspect variables, at the *'Statistically Designed Experiments'* stage, to narrow-down to the Red X. This limits the on-line testing which cause process disruption [32]. Once the Red X has been identified the process is optimised before implementing control measures to sustain process gains.

This narrow-down concept is achieved by using objective analysis instead of subjective opinion which is seen as *'guessing'* by Shainin, see [51]. Therefore, the strength of the Shainin System is in its thorough exploration of a quality problem [49]. The system also achieves results by conducting experiments with limited sample sizes, making it suitable in a low-volume paradigm. However, the methodology divides opinion, and these extreme views are considered in the next two sections.

### 2.3.3 Successes

Although the academic literature on the Shainin System is limited, a key strength that has been identified is in its initial analysis of a problem. Steiner et al. [32] supports this, stating *'the algorithm is very strong for the diagnostic journey'* and De Mast [16] identifies that the Shainin System is strong in the *'exploration phase'* of a problem. Beyond this academic view there are also successful practical implementations.

A major success of the Shainin System, like Six Sigma, is attributed to Motorola and their winning of the first Baldrige Award in 1988 [52]. Other case studies include: Lucas Engine Management Systems, who saved $150,000 per annum after applying the Shainin System to reduce variation in a electromechanical automotive component [51]; USS-Posco, who made *'Monthly savings...in the six figures'* after applying the Shainin System to identify a Red X to reduce the quantity of Zinc used to produce Zinc-plated rolled steel sheets [51]; Jegadheeson et al. [53] reported significant improvements in the capability of a stator manufacturing process, the stators were a component in an auto-electric alternator assembly; In another study of an alternator assembly, Shanmugam and Kalaichelvan [54] applied the Shainin System to reduce rejections by 50%; Mooren et al. [55] utilized the Shainin System to identify the root cause of premature drill wear out, this reduced process down time which had a financial

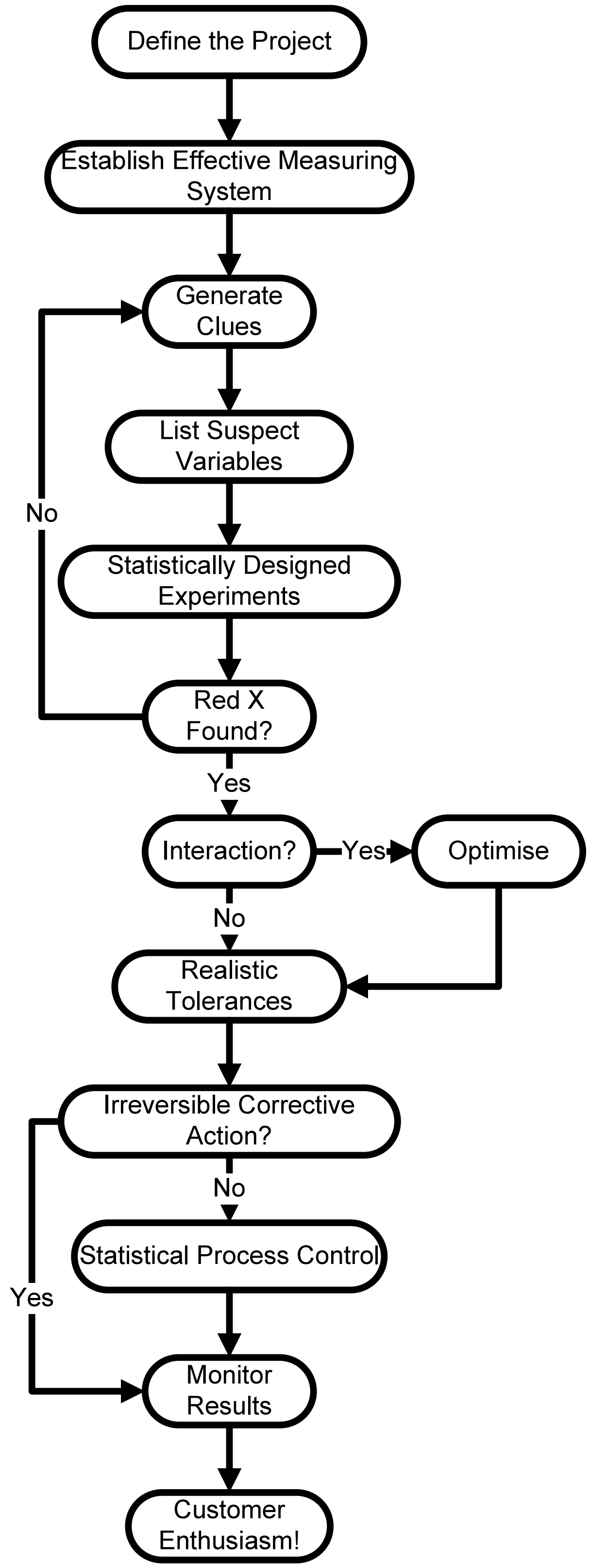


Figure 2.2: Reproduction of the Shainin System algorithm [32].

implication on the cost of a final product. Despite these successes, the Shainin System has proven to be controversial and this is explored in the next section.

### 2.3.4 Issues

There has been little peer review work for the Shainin System and its methods, and the literature that exists is divisive. Senapati [33] states that this controversy is a result of the systems development by consultants, rather than through transparent application of well described theories. An example of this controversy is with the most complete description of Shainin tools by Bhote [56] and Bhote and Bhote [52]. These texts are heavily criticised by Hockman [57] and Ziegel [58] for being self-promotional and for their dismissal of classical DOE techniques. However, a more balanced view of these texts by Steiner et al. [32], identifies that they present many useful Shainin techniques, but do them a *'disservice since the hyperbole hides many of the genuinely useful ideas'*. It is noted that much of the academic criticism of the Shainin System focuses on individual tools employed within the algorithm rather than with the algorithm itself. The criticism of tools is further explored in chapter 3.

## 2.4 Hybrid Process Improvement Methodology

### 2.4.1 Context

This section outlines a hybrid Six Sigma approach which incorporates Shainin concepts such as *'narrowing-down'* on a root cause, outlined in Cox [59]. The philosophy of hybridising two seemingly competing process or quality improvement methodologies is not new. As detailed in section 2.1, in industry there is widespread use of Lean Six Sigma or Lean Sigma programmes, which are amalgams of Lean and Six Sigma methodologies. This merger of Lean Six Sigma has occurred despite some stark fundamental differences in approach. An example of this is the difference in implementation were Lean focuses on bottom-up application, Six Sigma uses a top-down application. In this context, the integration of Six Sigma and the Shainin System should be seen as a smaller leap than that of Lean and Six Sigma. The idea of integrating Shainin System elements and tools in to other process improvement methodologies is supported by [32], who states: *'[Shainin tools] should be incorporated into other process improvement methodologies'*. Further to this, Sharma and Chetiya [60] provide case studies in an automotive and general lighting lamp manufacturing company, where Shainin system tools were successfully implemented in a Six Sigma project. Although they do not build an explicit framework for their implementation.

Six Sigma and the Shainin System have strong methodological similarities in their approaches. A detailed exploration of the similarities and differences between the Shainin System and Six Sigma is be found in De Mast et al. [49]. By comparing Figures 2.1 and 2.2, it can be seen that the structure of the Shainin System and Six Sigma have many similarities. Both methodologies go through five main stages:

1. Both start by defining a quality problem.
2. The measurement system is then validated through the an *'Establish Effective Measuring System'* step in the Shainin System and the Measure phase in Six Sigma.
3. Exploration of the quality problem is achieved with a *'Generate Clues'* step in the Shainin System and the Analyse phase in Six Sigma.
4. The process is then optimised, with an explicit *'Optimise'* step in the Shainin System and Improve phase in Six Sigma.
5. Finally both then monitor the improved process through a *'Statistical Process Control'* step and Control phase, respectively.

These similarities extend beyond there systematic structures. Both methodologies use rhetoric which is comparable; when referring to measured outputs the Shainin System talks about Green Y's where as Six Sigma refers to CtQ's. When analysing inputs which affect a process; the Shainin System labels the important factor as the Red X and Six Sigma terms this factor as the root cause of variation.

The key difference between these problem solving approaches is in the exploration of a quality problem [16]. Six Sigma aims to find root causes or suspect variables in a manner which has been identified as problematic. The Shainin System narrows-down to suspect variables by objectively ruling out variables which have an insignificant effect on a final Green Y [16, 31, 32]. Hence, the Shainin System eliminates unimportant factors from the search for the Red X using quantitative analysis, Six Sigma identifies root causes with a qualitative approach before validating this with a quantitative analysis. It is shown in the following section how Shainin System elements can be explicitly integrated into the overall Six Sigma approach to strengthen the exploration of a quality problem.

### 2.4.2 Structure

As identified in the previous section, Six Sigma lacks objectivity in its early Analyse phase, but it is in this exploration of quality problems where Shainin is particularly strong. In an attempt to improve this situation, the hybrid Six Sigma framework outlined in Figure 2.3 was proposed in [59].

This hybrid Six Sigma methodology still holds the DMAIC structure at its core and implements the Six Sigma 12-step strategy to obtain useful process metrics. It aims to reinforce the need for objectivity in the Analyse phase by incorporating a *'Find the Signature'* and *'Narrow Down'* loop. This loop underpins the PROcess VAriation Diagnosis Tool (PROVADT) tool, which is developed in Chapter 4. The result of this review identified a Hybrid Shainin Six Sigma methodology as the most appropriate framework to guide operators through a process improvement project in a low-volume paradigm. Therefore, the remainder of this thesis develops statistically derived tools and techniques that fit within this hybrid Six Sigma methodology.

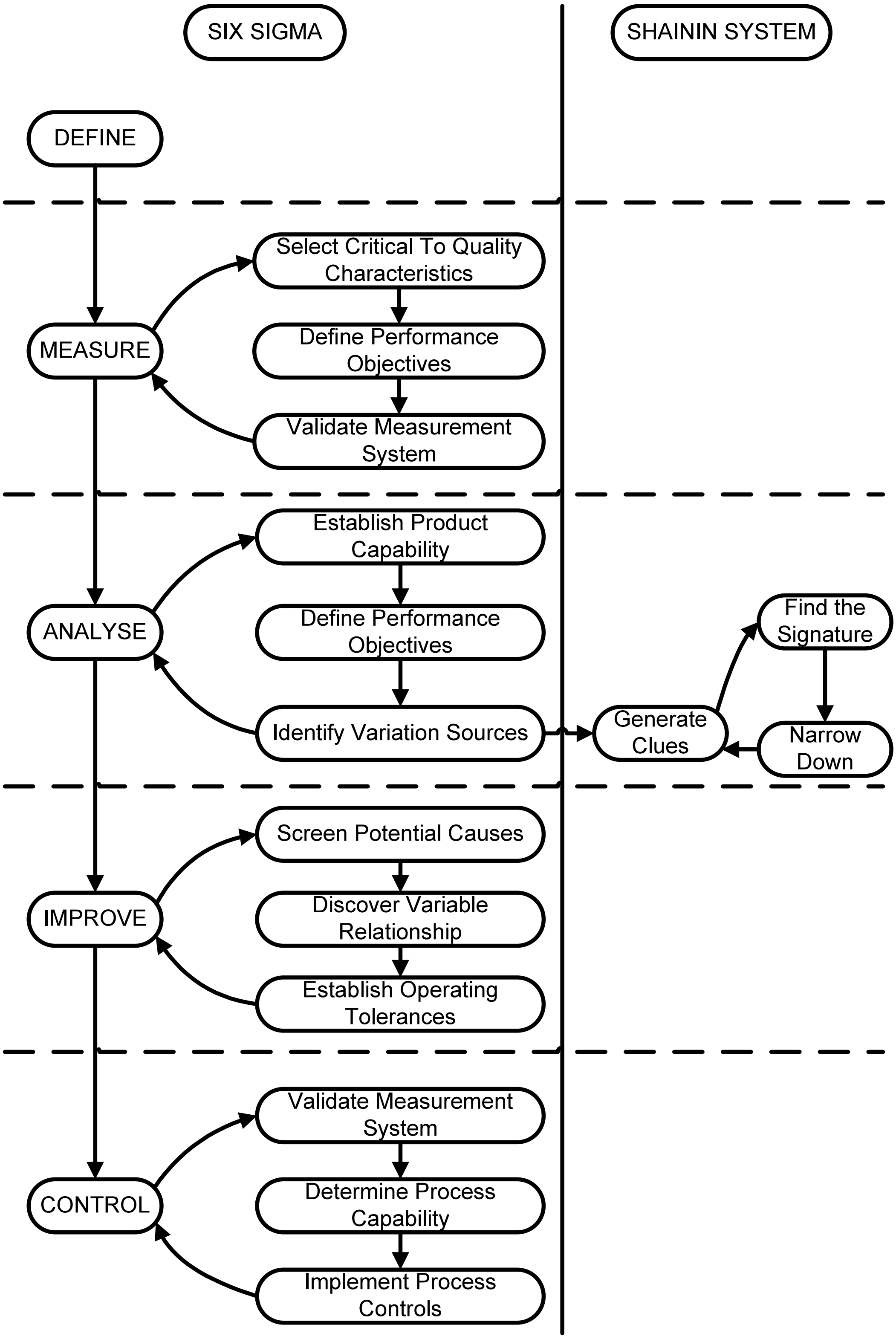


Figure 2.3: Reproduction of the Hybrid Six Sigma methodology [59].

## 2.5 Summary

In this chapter, a review of current process improvement methodologies was performed. This review was conducted in order to determine the most appropriate process improvement methodology to support process improvement projects in low-volume applications. This review assessed three approaches in particular: Six Sigma, the Shainin System and Hybrid Six Sigma. The results of this review identified the Hybrid Six Sigma methodology as the most appropriate for low-volume processes.

Six Sigma was identified as having numerous successful industry implementations, these successes are underpinned by the rigourous DMAIC cycle. However, it was shown in the literature that six Sigma is weak in the analyse phase or the ' *exploration*' of a quality problem, with a heavy utilisation of subjective analysis rather than objective analysis.

The Shainin System in comparison, was identified as being particularly strong in the ' *exploration*' of a quality problem. However, the literature that exists on the methodology is divisive. This is a result of its development and promotion by consultants, with the leading texts on the methodology being criticised by academics for being self-promotional and dismissive of classical DOE techniques.

To balance these two approaches the Hybrid Six Sigma approach was reviewed. This methodology is underpinned by Six Sigma's classic DMAIC cycle; however, it utilises the narrowing down approach from the Shainin System to strengthen the Analyse phase. Therefore, the remainder of this thesis will develop tools and techniques that sit within this Hybrid Six Sigma approach.

# Chapter 3

# Literature Review II - Process Improvement Tools



## 3.1 Introduction

In the previous chapter, a hybrid Six Sigma process improvement methodology was introduced. It was identified as the most suitable method to assist an operator problem solve in a low-volume production environment. This hybrid algorithm aims to go step-by-step through the traditional stages of the DMAIC cycle and collect Six Sigma metrics. The key addition to Six Sigma in the hybrid strategy, was the Shainin Y to X approach for narrowing-down to root causes of process variation. However, to apply this hybrid Six Sigma methodology in a low-volume production environment, process improvement tools are required. This chapter reviews the literature on process improvement tools suitable to complete three stages of the hybrid Six Sigma methodology, namely: Measure, Analyse and Control.

## 3.2 Measure

### 3.2.1 Introduction

In the Measure stage of a Six Sigma project, the defined problem is described in a measurable form and the current capability to perform these measurements is assessed [38]. This is achieved in three phases of investigation, as highlighted in Figure 2.3. These phases are

*Select CtQ Characteristics*, *Define Performance Objectives* and *Validate Measurement System* [37, 38, 61]. The first two phases are usually evident through engineering design drawing, Computer-Aided Design (CAD) models and tolerances, and the third stage, *Validate Measurement System*, is achieved through an investigation know as Measurement System Analysis (MSA).

MSA investigations are used to establish that the variation in the measurement system is at an acceptably low level. Each type of MSA makes different assumptions about known factors, in order to determine if a certain category of measurement system variation exists. An assumption made by all MSA techniques reviewed in this chapter is that the test is non-destructive and repeatable. Initial assumptions to validate a destructive test MSAs are found in De Mast and Trip [62], but destructive tests not addressed further as it is outside the scope of this review. The next section details the categories of measurement system error that variation can fall into, then section 3.2.3 details current MSA techniques.

### 3.2.2 Categories of Measurement System Error

The intension of this section is to give brief description of the types of variation that can exist in a measurement system. In general, there are five categories of measurement system error. These are repeatability, reproducibility, bias, linearity and stability errors [63].

**Repeatability** is the variation in measurements when repeatedly measuring one CtQ on the same part by one appraiser. This is represented in Figure 3.1(a). **Reproducibility** is the variation in the mean measurement of the same sample between appraisers. This is shown in Figure 3.1(b). **Bias** is the difference between the observed mean value of repeated measurements and a known reference value. This effect is seen in Figure 3.1(c). **Linearity** is the difference between bias values at different values within the operating range of the measurement system. This effect is seen in Figure 3.1(d). **Stability** is the total variation in a measurement system on the same sample over an extended time period. This is highlighted in Figure 3.1(e).

The following section 3.2.3 outlines techniques currently used to identify with statistical confidence if any of these error types exist in a measurement system.

### 3.2.3 Measurement System Analysis (MSA) Techniques

#### 3.2.3.1 Type *I* Gage Study

A type *I* gage study is a tool used to evaluate the repeatability and bias of a measurement gauge. The version outlined in this section for discussion is detailed in Minitab Technical Support Document [64], but a similar experimental method is described in AIAG [63]. It achieves this by measuring a single reference sample repeatedly, with a single appraiser. This analysis effectively focuses on the gage rather than the entire measurement system [64]. In essence it is a check of the gage's calibration. Facilities which use measurement gages and con-

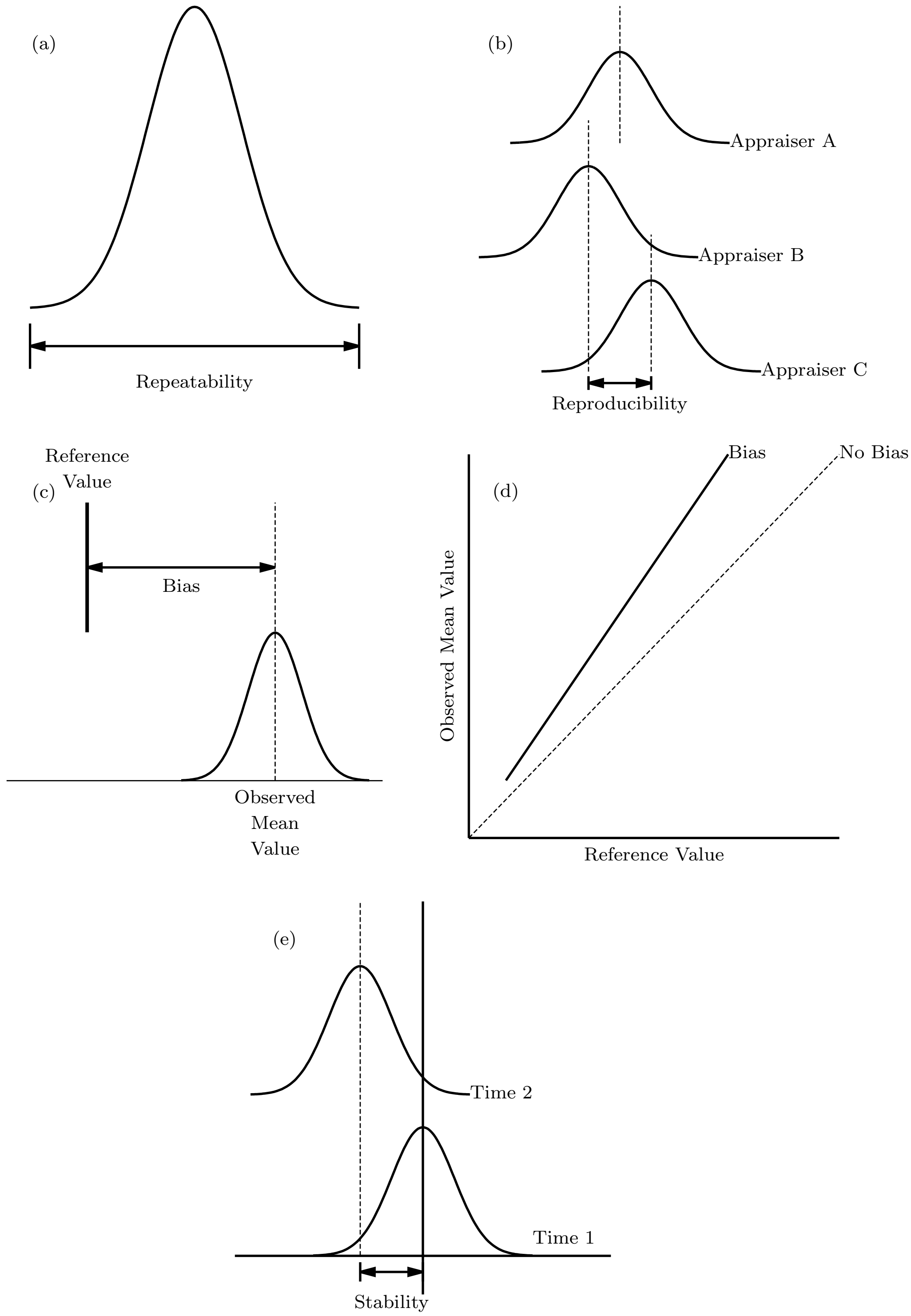


Figure 3.1: Measurement system error types [63]: (a) repeatability; (b) reproducibility; (c) bias; (d) linearity; (e) stability.

form to quality standards, such as BS EN ISO 9001:2008 [65] or ISO/IEC 17025:2005(E) [66], need to regularly check gage calibration. A part of this process is evaluating repeatability and bias, a specific example of calibrating weighing machines is given by [67].

A type $I$ study assumes there is a reference sample with a known CtQ. This sample is then measured repeatedly using the same measurement gauge and appraiser. It is typical that in excess of twenty-five measurements are made to allow for the estimation of variance. Once the repeat readings are taken, the measurement's repeatability is represented using the repeatability metric, $C_g$, which is calculated by:

$$C_g = \frac{\frac{P}{100} \cdot (U - L)}{n_m \cdot \sigma_m}, \tag{3.1}$$

where, typically, the percentage of the CtQ tolerance, $P = 20\%$; the number of standard deviations, $n_m = 6$; standard deviation of repeated measurements, $\sigma_m$; the difference between $U$ and $L$ is the tolerance range. The measurement repeatability and bias are represented together using the repeatability and bias metric, $C_{gk}$, by:

$$C_{gk} = \frac{(\frac{P}{100} \cdot (U - L)) - |\bar{X}_m - X_{ref}|}{\frac{n_m}{2} \cdot \sigma_m}, \tag{3.2}$$

where, the mean of all measurements, $\bar{X}_m$, and the reference value, $X_{ref}$. For $C_g$ and $C_{gk}$ a value of 1.333 or greater is required to pass this analysis. If there is a difference between the $C_g$ and $C_{gk}$ this indicates that there is a constant bias in the measurement gauge.

This technique is able to identify bias and repeatability of a measurement system. However, it relies on the assumption that a reference sample is available, which in a low-volume production environment is unlikely to be available. This is due to the high-variety and -value of products made by a single process, making the maintenance of reference samples for each product expensive and impractical. Therefore, the type $I$ gage study is unlikely to be suitable in low-volume production environment.

#### 3.2.3.2 Stability Study

The measurement stability study is a tool used to identify variation in a measurement system over long periods of time. A procedure to achieve this is documented in AIAG [63]. As in the case of the type $I$ gauge study, the stability study requires the availability of a single reference sample or the retention of a production part. This reference sample or retained part as then measured repeatedly on a periodic basis. For example, a reference sample might be measured three times, once per day. Although the number of repeats and frequency of testing can be adjusted to suit the operating conditions of the measurement system. The results are then monitored on a $\bar{X}\&R$ or $\bar{X}\&s$ control chart[1], to identify if a statistically significant change has occurred in the measurement system over time.

[1]Control charting methodology is examined further in section 3.4.

This technique clearly only identifies the stability component of measurement system variation. It also suffers the same shortcomings as the type $I$ gage study with respect to low-volume production, that is it is unlikely for there to be reference samples or retain production parts available for measurement.

#### 3.2.3.3 Type II Gage Study or Gage Repeatability and Reproducibility (R&R)

The Gage R&R study is a tool typically used in Six Sigma to validate a measurement system. It aims to statistically quantify the repeatability and reproducibility of a measurement system. To achieve this, it is assumed that when products are measured repeatedly the test itself does not damage the product. When these products are measured, the total variation, $\sigma_T^2$, recorded is: the sum of the part-to-part variation, $\sigma_P^2$ and the measurement repeatability and reproducibility variation, $\sigma_{RR}^2$:

$$\sigma_T^2 = \sigma_P^2 + \sigma_{RR}^2. \tag{3.3}$$

In order to assess the magnitude of $\sigma_{RR}^2$, a Gage R&R study is a measurement system analysis tool commonly used. It is classically used at the measure phase within DMAIC to statistically validate the repeatability variation, $\sigma_E^2$, and reproducibility variation, $\sigma_A^2$, of a measurement system, therefore:

$$\sigma_{RR}^2 = \sigma_E^2 + \sigma_A^2. \tag{3.4}$$

The $\sigma_E^2$ variation is the difference between repeated measurements by the same appraiser. The ability of an appraiser to provide consistent measurement data is important in the improvement of any process. $\sigma_A^2$ variation is the difference between different appraisers measuring the same CtQ under the same conditions [63]. The total Gage R&R variation is then expressed as a percentage of $\sigma_T^2$, as follows:

$$\%\sigma_{RR} = \frac{\sqrt{\sigma_E^2 + \sigma_A^2}}{\sqrt{\sigma_T^2}} \times 100. \tag{3.5}$$

The precision-to-tolerance ratio, $P/T$, is expressed as a percentage of the total Gage R&R variation against the design tolerance, $U - L$, as follows:

$$P/T = \frac{\sqrt{\sigma_E^2 + \sigma_A^2}}{U - L} \times 100. \tag{3.6}$$

In both equations (3.5) and (3.6), $\%\sigma_{RR}$ and $P/T$ should be less than 10% for the measurement system to be deemed adequate. If it is between 10% and 30% then it is considered marginal and anything above 30% is inadequate [63].

Ideally both $\%\sigma_{RR}$ and $P/T$ metrics should determine the measurement system to be adequate. However, there are two circumstances where one of these metrics may deem the measurement system to be inadequate but it is still appropriate to continue the use of this measurement system. The first case is where there is a small amount of part-to-part variation. In this example the denominator of the equation (3.5) is minimised, which means the $\%\sigma_{RR}$ can be greater than 10% but the $P/T$ may still be under 10%. This situation shows that the measurement system is able to detect large changes in the process with respect to the CtQ tolerance. However, due to the extremely high capability of the process the measurement system is unable to distinguish between parts that fall under the normal part-to-part variation. The second case is where there is a large amount of part-to-part of variation and there is a significant amount of $\sigma_{RR}^2$. In this example the $P/T$ can be greater than 30% but the $\%\sigma_{RR}$ is less than 10%. For this situation it is still possible to use the current measurement system to investigate process variation, but it would be desirable to obtain an improved measurement system for ongoing process measurements.

There are many approaches to both the design of a Gage R&R experiment and the analysis of its results. Gage R&R studies are usually performed with a balanced design, which means all appraisers repeatedly measure all samples parts the same number of times [63]. Although recent work by Stevens et al. [68] has found that unbalanced or augmented experiments can have substantial gains in efficiency when compared to balanced standard plans that use a similar number of measurements.

There is also a decision on whether to perform a fully-crossed or a nested Gage R&R design. In a fully crossed design all appraisers measure all the sampled parts, whereas in a nested design appraisers measure separate batches of parts. Although the nested design provides an experiment plan when the opportunities to measure parts is limited, it does confound batch-to-batch product variation with appraiser-to-appraiser measurement variation. It is therefore desirable to use a fully-crossed design where possible.

Traditionally, sixty measurements are used to obtain a Gage R&R. Following the guidelines by the AIAG [63], these measures should be taken over samples representing the full range of product variation, and with at least two repeats by at least two appraisers. The ability to expand a Gage R&R by increasing the number of repeat measurements, makes it usable in a low volume production environment where samples are limited. However, the method only assesses the repeatability and reproducibility of a measurement system. Additional experiments are required to assess other error types, such as linearity and bias.

#### 3.2.3.4 Isoplot

An Isoplot is a tool associated with *'establishing a measurement system'* in the Shainin System. It provides a graph that is used to identify the scale of variation in a measurement system relative to the variation in the process [32]. This is achieved by two appraisers measuring the same CtQ on typically thirty products [32, 68]. As in Gage R&R methodology, appraiser means an alternative measurement system; this can be either different operators using the same measurement equipment or the same operator using different measurement equipments. The first fifteen products are measured by appraiser 1 first, then by appraiser 2. The second fifteen products are measured in the opposite order. From the results the measurement system variation is displayed graphically against process variation, to see which has the greatest effect. Figure 3.2 shows examples of Isoplot results seen when different error types are present.

In Figure 3.2 the common format of an Isoplot is presented. This shows that the measurements of a products CtQ from Appraiser 1 are plotted against the results from Appraiser 2. A 45° line is drawn from the origin to represent a perfect measurement system.

In Figure 3.2(a) all thirty measurements are on the same point on the 45° line. This indicates that the measured CtQ on all thirty products is identical with no variation in the measurement system.

In Figure 3.2(b) the thirty measurements are spread along the 45° line. This variation along the 45° line indicates the variation between the measured CtQs of the thirty products. The fact that these measurements are all on the 45° line shows that there is no variation in the measurement system, i.e. there is no difference between Appraiser 1 and 2 measurements.

In Figure 3.2(c) the thirty measurements are spread both along and perpendicular to the 45° line. As in Figure 3.2(b), the variation along the 45° line indicates the variation between the measured CtQs of the thirty products. However, the variation perpendicular to the 45° line shows there is variation in the measurement system, i.e. there is a difference between Appraiser 1 and 2 measurements of the same CtQ. If the ratio between the variation sources is such that the process variation is at least five times greater than the measurement variation, the measurement system is said to be valid with 98% confidence [52].

In Figure 3.2(d) the thirty measurements are grouped parallel to the 45° line. This displacement from the 45° line, means there is a bias between the two appraisers. An angle other than the 45° indicates a proportional error meaning non-linearity in one of the appraisers.

In Figure 3.2(e) the results are split into two distinct groups either side of the 45° line. This occurs when the measurement system itself has an effect on the results. For example, the second test is always less than the first, due to the product being damaged by the measurement system. This means that an Isoplot checks that a test is repeatable and non-destructive.

This section shows that an Isoplot can visually display measurement variation, bias and linearity errors, as well as checking the repeatability. The issue with this test is that the displayed measurement variation is a confounding of the measurement systems repeatability and reproducibility. This is fine if the measurement variation is small enough to validate the measurement system. However, if there is excessive measurement variation, further investigations are required to determine if this is a repeatability or reproducibility error.

### 3.2.4 Summary

In this section the five types of measurement system error were observed. Measurement system analysis techniques that are currently in use in industry and in the literature were

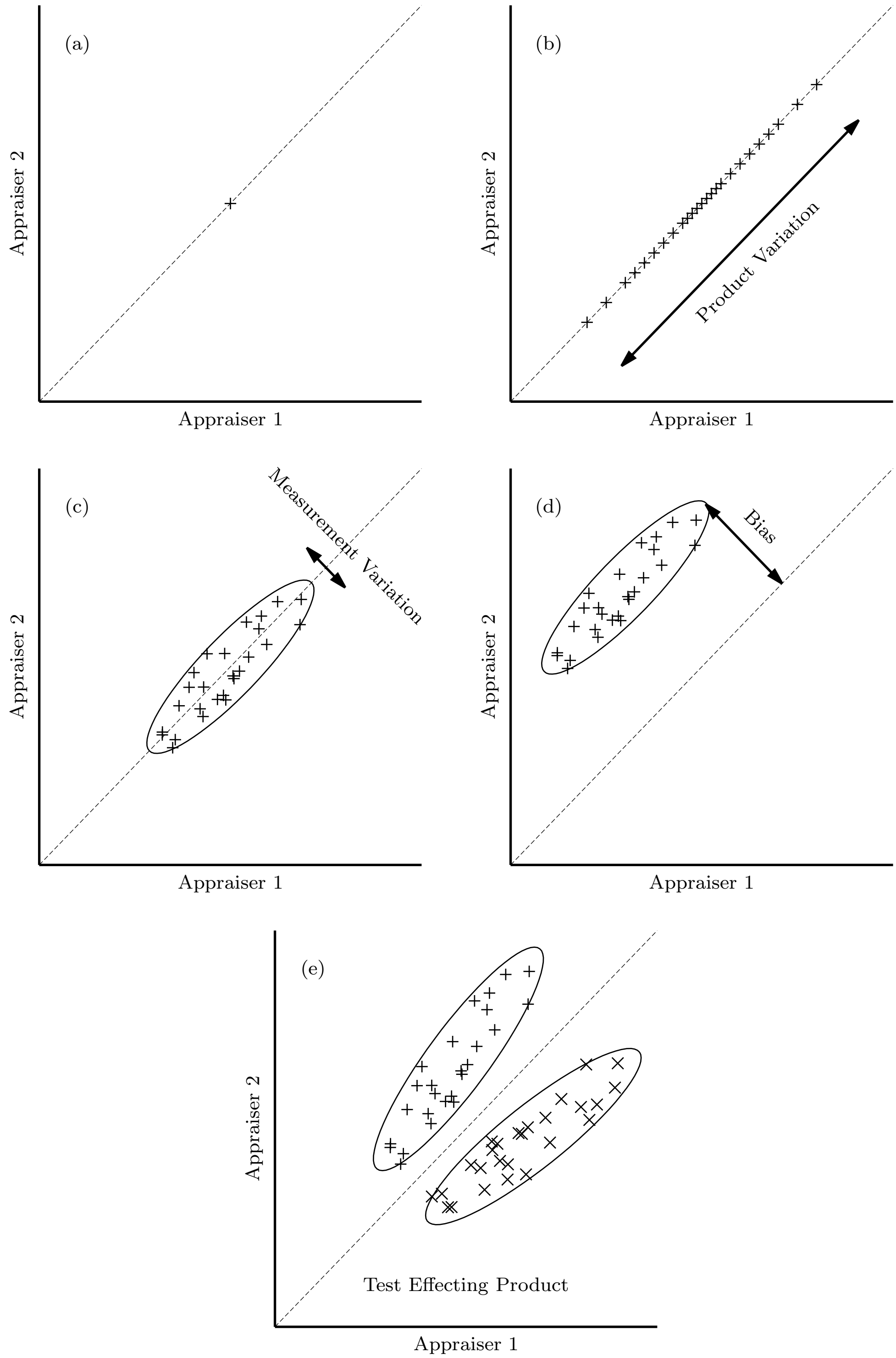


Figure 3.2: Examples of Isoplot Results: (a) A perfect product and measurement system; (b) Variation in the 30 products and a perfect measurement system; (c) Variation in the products and the measurement system; (d) A bias in one appraisers measurement; (e) The test is effecting the product.

reviewed. Their ability to detect types of measurement system error was identified and their ability to be applied in a low-volume environment was assessed. In the next section, analysis techniques are reviewed to identify methods that can progress a hybrid Six Sigma project through the analyse phase in a low-volume production environment.

## 3.3 Analyse

### 3.3.1 Introduction

In the analyse stage of a Six Sigma project, the process is analysed to determine its current performance and identify causes of variation in a process. This is achieved in three phases of investigation, as highlighted in Figure 2.3. These phases are *Establish Product Capability*, *Define Performance Objectives* and *Identify Variation Sources*. The first phase, *Establish Product Capability*, is achieved through a series of statistical experiments known as process capability studies, these will be reviewed in section 3.3.2. The second phase, *Define Performance Objectives* is not reviewed in this thesis, as it is a function driven by design objectives that production has to match. The third phase, *Identify Variation Sources*, is achieved in classic Six Sigma through a series of subjective analysis and classical statistical tests. However, within the hybrid Six Sigma's methodology this phase is reinforced with the Shainin Y to X approach to experimentation and the associated tools. Therefore the tools associated with the hybrid Six Sigma *Identify Variation Sources*, contains techniques from both classic Six Sigma and the Shainin System, which is discussed in section 3.3.3.

### 3.3.2 Establishing Product Capability

#### 3.3.2.1 Process Capability Indices

The statistical tool that is used in Six Sigma projects to establish product capability is the process capability index. A survey study of companies using process capability indices was performed by Deleryd [69]. It was found that the main advantage of using these metrics was they provide an increased knowledge of the process and enabled fact-based decisions to be made about the progression of improvement projects. This study also found that companies believe the methods of the resource consuming and involving some difficult theory, which acted as a barrier to implementation. This perception of a method being resource consuming is a particular barrier to it being used in a low-volume manufacturing context.

Although there are few different process capability indices, in practice there are two which are most commonly used. These are process capability precision metric, $C_p$, and process capability precision and accuracy metric, $C_{pk}$. This section will review these two techniques further and examine how they can be applied in a low-volume context.

#### 3.3.2.2 Process Capability Index, $C_p$

To assess the total variation in a process as a ratio against CtQ tolerance, it is typical to use $C_p$. In order to calculate this metric, it is usual to collect and measure the CtQ of fifty consecutive products from a process. Then the process standard deviation, $\sigma$, of these measurements is used to calculate $C_p$ as follows:

$$C_p = \frac{U - L}{6\sigma}. \tag{3.7}$$

The $6\sigma$ term has been used historically, because this represent 99.7% of the process population. This index can only be used when a CtQ has a bilateral tolerance specified, i.e. both $U$ and $L$ are specified.

#### 3.3.2.3 Process Capability Index, $C_{pk}$

To assess the total variation in a process as a ratio against CtQ tolerance and mean deviation from design target, it is typical to use $C_{pk}$. In order to calculate this metric, it is usual to collect and measure the CtQ of fifty consecutive products from a process. Then the process mean, $\mu$, and $\sigma$ of these measurements is used to calculate $C_{pk}$ as follows:

$$C_{pk} = min\left[\frac{U - \mu}{3\sigma}, \frac{\mu - L}{3\sigma}\right]. \tag{3.8}$$

It is also common to use this index when a CtQ has a unilateral tolerance specified, i.e. only one of $U$ or $L$ is specified.

#### 3.3.2.4 Mini-Capability

It has been outlined in the descriptions of $C_p$ and $C_{pk}$ that fifty consecutive products are collected to establish the process capability. Deleryd [69] identified that companies found this resource consuming. This is a further issue in a low-volume manufacturing environment, where a production run of fifty units would be considered large. A solution to this is the application of mini-capability studies [10]. A mini-capability study calculates the $C_p$ and $C_{pk}$ indices using a sample size as small as ten products.

This method provides a useful approach to capture a snapshot of current performance and act as a baseline for an ongoing process improvement project. However, it would not be recommended to use this mini-capability as the basis of SPC monitoring[2].

### 3.3.3 Identifying Variation Sources

#### 3.3.3.1 Root Cause Analysis Approach

It is common for traditional Six Sigma to use a root cause analysis approach for the identification of variation sources. This typically involves, reviewing process data, developing causal

[2]Control charting methodology is examined further in section 3.4.

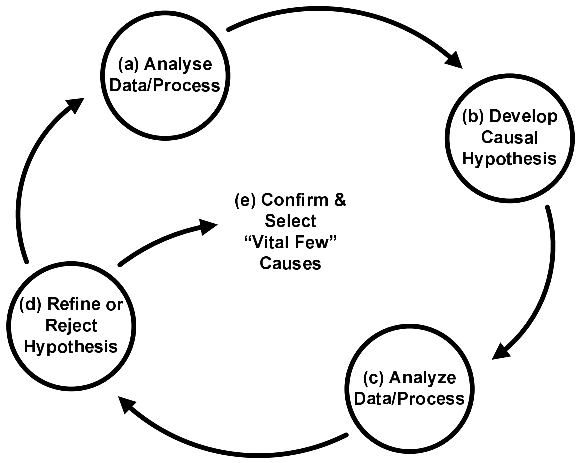


Figure 3.3: Reproduction of the root cause analysis cycle [24].

hypotheses and then performing statistical analysis to validate his hypotheses. An example of this approach is shown in figure 3.3.

The first stage of the root cause analysis cycle, shown in figure 3.3 (a), is to analyse data/process. This data is usually historical data collected on an ongoing basis, it is then reviewed using techniques such as scatter plots to correlate variations in the output with variations in the inputs. There is a risk with scatter plots, that correlations appear as a result of coincidence, or an unidentified factor [27]. This risk potentially leads to erroneous results. There is also the potential to simply miss a root cause from the analysis, if the correct data is not available. The scatter plot analysis also requires large datasets to provide confidence in the results, which may not be available in a low-volume situation. At the same time, the process is reviewed using Ishikawa diagram, cause-and-effect matrices and brainstorming [27, 39, 70]. These techniques aim to identify input variables that affect the output variable, however, they are highly subjective and reliant on expert opinion. To that end, these techniques are considered at best to be *'educated guesses'* [24] and at worst just a guess!

The purpose of these analyses is to generate *'causal hypotheses'* in the search for the root causes of variation. Once a small number of hypotheses have been developed, or potential root causes identified, further more rigourous statistical experimentation takes place to confirm or reject these hypotheses. The types of statistical experimentation classically used at this stage range from basic regression analysis and t-tests to more elaborate experiments such as ANalysis Of VAriance (ANOVA) and DoE. However, these techniques can be difficult to apply in low-volume situations, particularly if there are a large number of potential root causes incorporated in a hypothesis. For example, if there were five potential root causes and a full factorial DoE was applied there would be a minimum of $2^5 = 32$ experiments needed. In a low-volume environment there is limited opportunity to perform these experiments. This high number of forced experiments would difficult to achieve in a low-volume production enviroment. If it was achievable logistically, the products are of a high-value which may make the cost of sampling prohibited. Using less powerful screening techniques such as fractional factorials [48], reduces the numbers of experiments needed (as all combinations of experimental factors are not run) but at the expense of understanding higher order interaction effects [10]. However, the number of experiments required can still spiral out of control if there are multiple factors present. It has been noted that in a low volume manufacturing process, quality practitioners are increasingly leaning towards the use of subjective approaches to avoid the practical limitations of these types statistical analysis [1].

#### 3.3.3.2 Y to X Approach

In the hybrid Six Sigma approach, describes in section 2.4 and employed in this thesis, a Y to X approach to identifying variation sources is used. This approach is deployed by integrating the Shainin System *clue generation* phase into the *Identify Variation Sources*. The principal technique of the Shainin System *clue generation* phase is the Multi-Vari chart [32, 51, 52]. A Multi-Vari chart is a stratified experiment which identifies the signature of variation. The three strata of the study are within-piece, piece-to-piece or time-to-time variation. After a Multi-Vari chart is complete, each variation type is categorised as either the Red X, dominant variation; Pink X, secondary variation; or Pale Pink X, tertiary variation. Based on the Pareto principle of a vital few causes accounting for the majority of a problem, the study focuses the subsequent actions in an improvement project on eliminating the Red X variation.

To assess time-to-time variation, samples are collected from a minimum of five time periods which cover 80% of historical variation [52]. These time periods can be over chronological time periods such as hours, days or weeks; across operator shift periods; or production batches. Piece-to-piece variation is captured by collecting a minimum of three consecutive samples at each sampling time. Then, within-piece variation is accessed by measuring each sample repeatedly, at least three times. If the CtQ is a dimension, the repeated measures should be in different locations. For example the diameter of a shaft can be measured in different orientations to check ovality. If the CtQ is a property, for example electrical resistance, only one location can be measured.

Recent examples of its practical implementation, are given in Shanmugam and Kalaichelvan [54], Doganaksoy and Hahn [71] and Zaciewski and Németh [72]. The use of Multi-Vari charts within a Six Sigma framework to assist the analyse phase is illustrated in Goh [73] and Snee [18], for recent case studies see Cox et al. [74]. In either context, it allows a practitioner to narrow down to a few potential root causes of variation objectively [75, 76].

## 3.4 Control

### 3.4.1 Introduction

Process control procedures are decision algorithms used by process operators to signal when to adjust control parameters. These adjustments arise when it is deemed that there is either too much variation in the current production run or the process is off-target with respect to design features tolerances. In a Six Sigma project, process control procedures are implemented in the control phase to maintain a processes optimised performance. Six Sigma typically uses a statistical approach to process control known as Statistical Process Control (SPC), which was originally conceived by Walter A. Shewhart [77].

SPC is a collection of techniques that commonly use control charts to compare a processes current capability against its in-control capability. First, the stable variation, or *common cause* variation, in a process is established. The process is then regularly monitored to identify when either: $\mu$ has deviated significantly from $T$, or when $\sigma$ has increased significantly. Therefore, SPC techniques detect when significant changes in the $C_{pk}$ of a CtQ. If a change is detected, a quality improvement initiative is implemented to determine the *assignable cause* of the change.

Historically, this approach has been successful at monitoring high-volume discrete manufacturing processes. However, control methods for low-volume processes is a current concern [78]. As identified in chapter 1, these processes are highly capable over a small production run and the dominant source of variation is between batches. This is defined as a *set-up dominant process* [10]. In this environment, it is not possible to sample enough consecutive units to determine statistical control limits for the process. This has resulted in operators taking a subjective approach to controlling a process.

A further complication is the increasing complexity of products being manufactured. For example, if a product has thirty CtQ features, there is a need to produce thirty separate control charts to monitor each feature. This can become unmanageable for a process operator. Monitoring each individual CtQ independently is known as univariate SPC. There are techniques to monitor all CtQs on a single chart this is known as multivariate SPC. Therefore, this section of the literature review details the current approaches for low-volume process control currently used in industry, identifies approaches available in the literature and identifies the gaps in knowledge.

### 3.4.2 Industry Practice

In industries that operate low-volume discrete manufacturing processes, it is common to see process control procedures which are based on *rule of thumb* methods. A rule of thumb method can only loosely be described as SPC procedure, due to their extremely subjective approach which has a heavy reliance on operator experience and skill. Although it attempts to detect when a process is off-target to minimising process adjustments; it does this without a rigourous statistical framework. However, for set-up dominant processes, of either the univariate or multivariate case, these procedures are commonly used due to simplicity of implementation. This follows the trend of low-volume operations employing a subjective approach to quality improvement [1].

Two rule of thumb methods identified in use are 100% inspection and first-offs [79]. To apply First Off's, the first unit produced has its CtQ measured. The process is then adjusted towards $T$ of the CtQ tolerance; production is then allowed to continue. However, little statistically valid information can be gained from the measurement of one unit alone 100% Inspection is where all units are measured as they are produced. The process is then adjusted either after every unit measured off-target, or after every unit measured out of tolerance. This results in operators either *"hunting"* for the tolerance centre, or, even worse, allowing defects to occur. In either case, little statistical thought or method is applied to the data captured. These problems with rule of thumb methods were observed by Carter and Butler [80]. This case-study of precision turned components, reported that control of small batch production had *'a heavy reliance on the operators experience'*. Instances of variability in a process being induced by the operators making unnecessary changes, were reported.

### 3.4.3 Univariate Process Control

The first Statistical Process Control (SPC) methods were introduced by W.A. Shewhart whilst working at the Bell Telephone Laboratories, Inc. during 1920's and 1930's [81, 82]. These have been popularised in high-volume manufacture today through quality standards, such as ISO/TS 16949:2009 [83] and BS EN ISO 9001:2008 [65].

Shewhart SPC based methods are a parametric approach, which means distributional assumptions are made about the process. Its operation is classically in two phases. In phase I, the 'common cause' variation is quantified. This is the variation that is ever present and uneconomical to remove. After the process is running, phase II identifies when 'assignable cause' variation occurs. This type of variation is a result of changes to the process causing drift in process mean or excessive spread of variation.

A common univariate Shewhart SPC approach for a variable measure is the Mean and Range Chart ($\bar{X}\&R$ Chart) (for examples see [84, 85]). This is obtained by plotting on a chart, the estimated subgroup mean, $\bar{X}$, and the subgroup range, $R$, against capability based control limits. To calculate $\bar{X}$ and $R$, the measured CtQ value, $x$, of each part in the subgroup of sample size $n$ is measured and therefore:

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}. \tag{3.9}$$

$$R = max(X_i) - min(X_i). \tag{3.10}$$

To calculate control limits, in phase I of the procedure twenty-five subgroups of four or five units each are typically collected. The mean of the subgroup means, $\bar{\bar{X}}$, is computed using $\bar{X}$ of the number of subgroups, $m$, by:

$$\bar{\bar{X}} = \frac{\sum_{j=1}^{m} \bar{X}_j}{m}, \tag{3.11}$$

and mean of the subgroup ranges, $\bar{R}$, is calculated using $R$ of $m$ by:

$$\bar{R} = \frac{\sum_{j=1}^{m} R_j}{m}. \tag{3.12}$$

This enables control limits to be obtained from:

$$\begin{aligned} UCL_R &= D_4\bar{R}, \\ LCL_R &= D_3\bar{R}. \end{aligned} \tag{3.13}$$

$$\begin{aligned} UCL_{\bar{X}} &= \bar{\bar{X}} + A_2\bar{R}, \\ LCL_{\bar{X}} &= \bar{\bar{X}} - A_2\bar{R}. \end{aligned} \tag{3.14}$$

Where, the upper and lower control limits are $UCL$ and $LCL$ (with subscripts $_{\bar{X}}$ and $_R$ denoting control limits for a mean or range chart respectively). $D_3$, $D_4$ and $A_2$ are statistical constants, which allow the estimation of $3\sigma$ from the mean subgroup range and can be found in statistical texts[84, 86].

Then while a process is being monitored in phase II, if $\bar{X}$ and $R$ fall within their respective control limits, the process is said to be 'in-control'. If $\bar{X}$ or $R$ all outside their respective control limits, the process is said to be 'out-of-control'. Whereby, $\bar{X}$ falling outside its control limits indicates there is a significant statistical change in the process mean and $R$ falling outside its control limits indicates a statistically significant change in the process variation. An example $\bar{X}\&R$ Chart is shown in Figure 3.4.

However, the application of SPC for set-up dominant processes is problematic [87]. Due to the low-volumes of parts produced, quantifying the common cause variation in phase I is impossible. Production of a small batch would have long been finished before twenty-five subgroups of four or five sample parts were made. Hence, not enough samples are available to acquire an estimate of this common cause variation.

Methods have been introduced which operate on the principle 'study the process, not the product' [88]. The Difference Control Chart ($\delta$-Control Chart), plots the results of different product batches on the same chart, by measuring the deviation from their target [89–92]. Hence, a difference between $\mu$ and $T$, $\delta$-statistic, is charted instead of $\bar{X}$, but the two are related by:

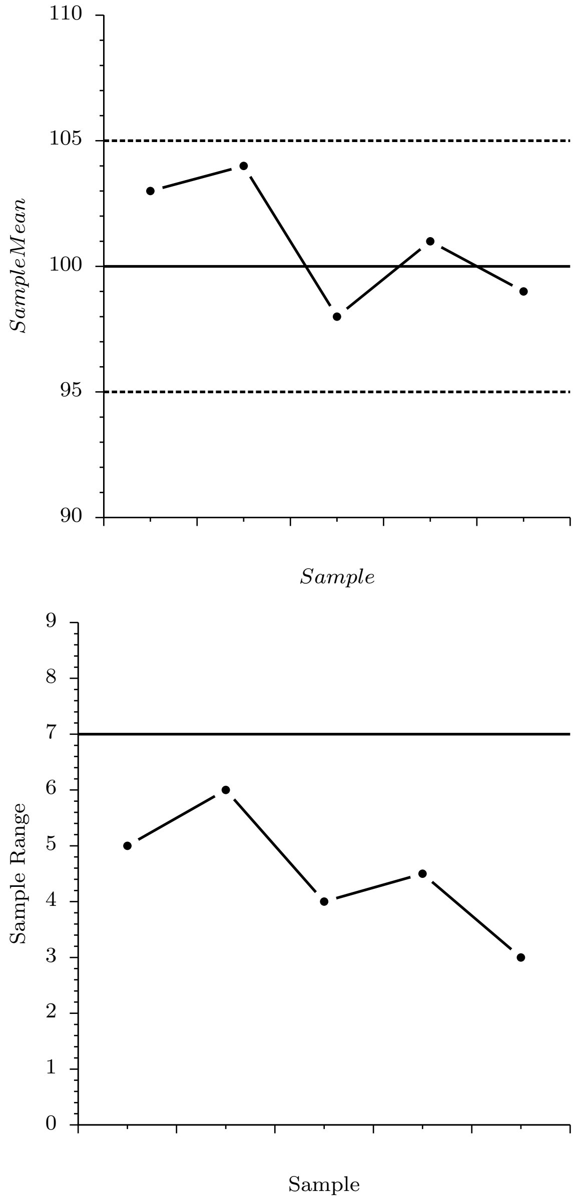


Figure 3.4: An example $\bar{X}\&R$ chart.

$$\delta = \bar{X} - T \tag{3.15}$$

However, there is still a requirement for a long start-up phase; making it unsuitable for set-up dominant processes. The $t$-chart is a modification of the $\delta$-chart approach, which is self-starting and eliminates the need for a start-up phase [93]. Although twenty five subgroups are not required to set control limits, the $t$-chart still collects data in subgroups. In small production runs, this could be near, or at the end of a run, meaning that set-up dominance is overlooked.

Attempts to optimise this have been made with the introduction of computer-aided SPC [94, 95]. This aims at scheduling the order of batches, so that similar products are manufactured in series, thus reducing the extent of set-up changes between batches. However, in a dynamic manufacturing environment where orders occur at short notice, this level of planning to support SPC implementation is not practical. The concept of self-starting SPC is also applied in the $Q$-chart [96].

An alternative form of SPC, is the Small-Batch $\bar{X}R$ Chart (SB$\bar{X}R$), which uses a five unit decision procedure [79]. This is similar to the Statistical Setup Adjustment technique [87]. Traditional $\bar{X}\&R$ methods wait for a subgroup of four or five units before a decision is made on the process capability. However, SB$\bar{X}R$ avoids waiting for a subgroup of five units by recalculating the statistical control limits of $\bar{X}$ and $R$ charts as the subgroup size increases from one to five units. Therefore, a process can be re-centred after one unit or too much process variation is detected after two units, overcoming the problem of waiting for a subgroup of five to indicate an issue. The recalculation of the statistical control limits is achieved by first estimating $\bar{R}_n$, using historical standard deviation, $\sigma$, as follows:

$$\bar{R}_n = d_2 \cdot \sigma, \tag{3.16}$$

where $d_2$ is a tabulated statistical constant, which can be found in statistical texts [84–86]. The value of $d_2$ is dependant on the sample size $n$ and is determined for sample sizes of $n = 2, 3, 4, 5$, because the control limit changes as the subgroup size grows. In an $R$-chart for SB$\bar{X}R$, the upper control limit, $UCL_R$, is calculated using the results from Equation (3.16), as follows:

$$UCL_{R,n} = D_4 \bar{R}_n, \tag{3.17}$$

where $D_4$ is another statistical constant which depends on the sample size $n$. Finally, the $\bar{X}$-charts upper and lower control limits, $UCL_{\bar{X}}$ and $LCL_{\bar{X}}$ respectively, are calculated for sample sizes of $n = 1, 2, 3, 4, 5$, using $\sigma$ and the estimated process mean, $\bar{X}_n$, as follows:

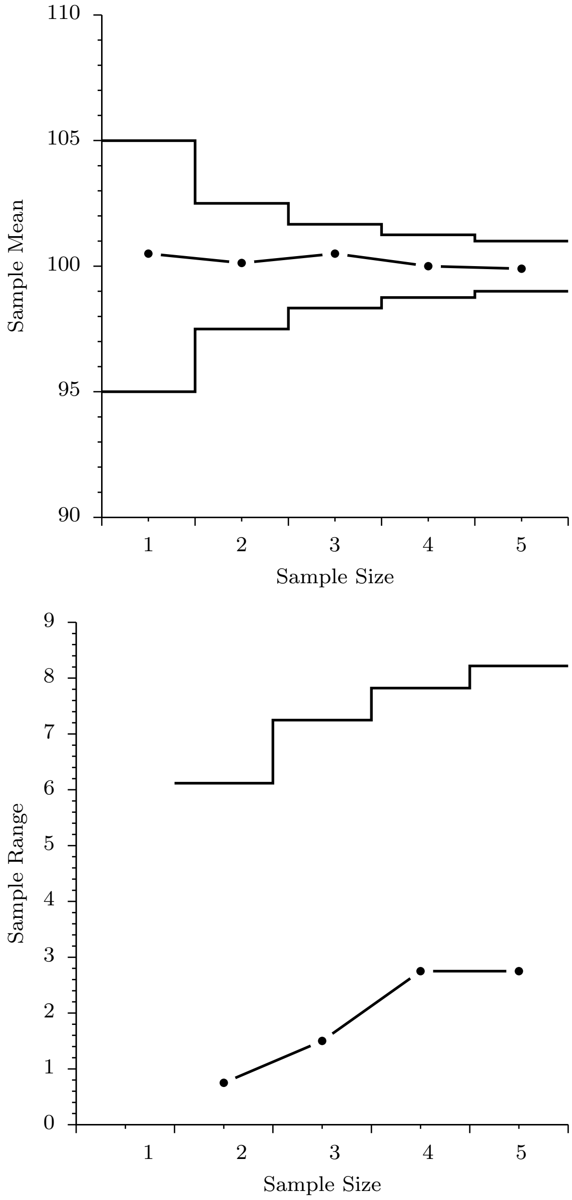


Figure 3.5: An example Small-Batch $\bar{X}$ and $R$ chart.

$$LCL_{\bar{X},n} = \bar{X}_n - 3\frac{\sigma}{\sqrt{n}}. \tag{3.18}$$

$$UCL_{\bar{X},n} = \bar{X}_n + 3\frac{\sigma}{\sqrt{n}}. \tag{3.19}$$

An example SB$\bar{X}R$ chart is shown in Figure 3.5. This figure demonstrates the characteristic narrowing of control limits on the mean (or $\bar{X}$) chart and widening of control limits on the range (or $R$) chart. The means and ranges of the progressively bigger subgroup size are plotted with points. In this example all sample means and ranges fall within the control limits. A significant disadvantage with this method, is the necessity to estimate the standard deviation of the process based on subjective assumptions, engineering knowledge, or possibly scarce surrogate/historical process data. Also, the need for constant recalculation of statistical control limits has the potential for error when applied by operators with little background in statistics.

An alternative to the previously described parametric forms of SPC, is to take a non-parametric approach. Non-parametric forms of process control make no distributional assumptions about the process. Instead, they base control limits around design targets and tolerances. They use sequential sampling and decision rules derived from probability theory, to ensure that a process is performing to a minimum standard.

Set-Up Verification [97], or Acceptance Control Charting (ACC) [86, 98], is a univariate technique which bases its control limits on the design target, $T$. Acceptance control limits are drawn as a fixed deviation from the nominal based on an estimate of the minimum process standard deviation $\sigma$, obtained from a sample of size $n$ and using the statistical constant, $Z_\alpha$. $Z_\alpha$ is derived using a predefined acceptable risk of false fail level, $\alpha_q$-risk and using $1 - \alpha_q$ as the confidence level from a standardised normal table, see [86]. If any sample mean above the upper or below the lower acceptance control limit is observed, a process is deemed as non-acceptable. As with SPC approaches, the ACC method, as described in [98], is slow to respond to off-target processes as it depends on the collection of a subgroup of samples. In the example where $n = 5$, five out-of-specification products could be manufactured before a change to the process is made.

A different, non-parametric approach to control for a set-up dominant process, is stage 1 validation of PRE-Control (PC) [10, 52, 84, 99–106]. Unlike classic approaches to SPC, PC makes no assumption of the underlying distribution of a process. Instead, its objective is defect prevention using a simple algorithm to ensure with 2% $\alpha_q$-risk any validated set-up has a minimum $C_{pk} = 1.333$ [52].

PC uses a traffic light system to divide the design tolerance of a measured CtQ design feature. The central region covering 50% of the design tolerance is designated as the Green Zone. The regions covering 25% of the tolerance, between the Green Zone and the design tolerance limits are the Yellow Zones. The regions outside the tolerance limits are the Red Zones.

Sampled parts have their CtQ categorized as Green, Yellow or Red. A Red sampled part signals that the process is off-target and an action should be taken to stop and adjust it. Two consecutive Yellow parts from the same Yellow Zone signal that the process is off-target, and an action should be taken to stop and adjust it. Two consecutive Yellow parts from opposite Yellow Zones signal the process dispersion is too great and an action should be taken to stop and investigate the root cause of variation. Five consecutive Green parts demonstrate the process is capable and it's allowed to continue.

PC's use of sequential sampling and control limits based on specification rather than process performance has many critics, e.g. [107–109]. These critics point out that PC is inferior to SPC as it has a constant risk of unjustifiably tampering with a process, thus, increasing variability. Using only five measurements to validate a set-up is insufficient and

PC has an inability to determine statistical control of a process. These points are countered by [104, 110], arguing that for PC five measurements are adequate given the different purposes of PC and SPC; waiting for twenty five subgroups to validate a set-up has the potential to allow many defects as it is difficult to determine statistical control of a process, even with SPC, due to the dynamic nature of manufacturing variation. However, a balance review of the method points out situations were PC has merit [99]. One example is machining operations, were an operator is able to change a process setting that directly affects the output CtQ being monitored. In this case, it is normally less expensive to make an adjustment than to produce a defect and it is this situation that operators of set-up dominant processes typically face.

A full derivation of probability of qualifying a process using a generalised form of PC is given by San Matias et al. [102]. In this analysis it is shown that three factors affect the probability of qualifying ($P(q)$). They are the number of yellow units to signal an issue ($t$), the number of green units to qualify a valid process ($k$) and the width of the green region ($G_U - G_L$). The width of the green region directly affects the probability of a yellow ($P(y)$) or a green ($P(g)$) unit occurring. These probabilities, along with the number of yellow units to signal an issue and the number of green units to qualify a valid process can be used to calculate the probability of qualifying a process, as follows:

$$P(q) = P(g)^k \cdot \frac{\sum_{h=0}^{t-1} P(y)^h}{1 - \sum_{h=1}^{t-1} P(y)^h \cdot \sum_{m=1}^{k-1} P(g)^m}. \tag{3.20}$$

Using this equation, a traffic-light control chart can be developed to suit specific needs. For example, if a chart was to be developed that maintained the rules of Pre-Control, i.e. $t = 2$, $k = 5$ and $P(q) = 0.98$, the equation would need to be balanced by adjusting $P(y)$ and $P(g)$. To achieve the correct adjustment of $P(y)$ and $P(g)$, the minimum capability the process is required to conform to is selected to determine the width of the green region. Using the Pre-Control example, the process is required to have a minimum $C_p \geqslant 1.333$ with an assumed gaussian distribution. Therefore, the minimum standard deviation, $\sigma$, is calculated by:

$$\sigma = \frac{U - L}{6C_p}. \tag{3.21}$$

Using this value of $\sigma$, $P(g)$ and $P(y)$ is calculated as follows:

$$P(g) = \Phi\left(\frac{G_U - T}{\sigma}\right) - \Phi\left(\frac{G_L - T}{\sigma}\right), \tag{3.22}$$

$$P(y) = \left[\Phi\left(\frac{U - T}{\sigma}\right) - \Phi\left(\frac{G_U - T}{\sigma}\right)\right] + \left[\Phi\left(\frac{G_L - T}{\sigma}\right) - \Phi\left(\frac{L - T}{\sigma}\right)\right]. \tag{3.23}$$

The Pre-Control green boundary of 50% of the design tolerance, is derived by balancing equations 3.20-3.23 with $t = 2$, $k = 5$, $P(q) = 0.98$ and a minimum $C_p \geqslant 1.333$. This confirms the result for $P(q)$ in [52].

A significant issue with PC, is that as the variance in a process reduces a greater percentage of off-target processes are qualified. In fact as the $C_p$ of a process approaches 2.33, more than 98% of product will qualify stage 1 validation, even if the process mean is two standard deviations off-target, as it's $C_{pk}$ will be equal to 1.33. As the accuracy of precision processes increases, maintaining a $C_{pk}$ of 1.33 is no longer adequate to keep the process on target. Therefore, by defining parameters of $G_U$, $G_L$, $t$, $k$, $P(q)$ and a minimum $C_p$, new Pre-Control variants are derived. San Matías and Giner-Bosch [111] suggests optimising these parameters for different Pre-Control applications using a Mixed-Integer Nonlinear Programming (MINLP) approach. Giner-Boscha and San Matías [112] takes this work further, producing optimised control plans based on defining one criteria of performance.

### 3.4.4 Multivariate Process Control

The SPC methods presented so far are all univariate approaches. When a set-up dominant process is multivariate, i.e. a process that is making a part with multiple CtQs, univariate approaches become inadequate [113]. Classic multivariate SPC approaches, such as Hotelling's $T^2$-test, [114], suffer the same issues as univariate SPC. For example multivariate SPC still needs a start-up period to estimate the process variation. Subgroups of data need to be collected before actions are taken and there is a potential need for constant recalculation of control limits. Further, classic multivariate SPC approaches are generally underpinned by the assumption of multivariate normal distribution which, in the context of a dynamic manufacturing environment, may not be valid [113].

A further problem is extrapolating a non-parametric method for the multivariate case. It is common for operators to apply a univariate PC chart on each design feature of a multivariate process, which can be time consuming to chart and analyse results. Attempts to define *'multivariate PC'* are presented in [115, 116]. However, these are multivariate versions of stop-light control as they define there zones based on estimated process performance, which requires an assumption of normality and an estimate of variation based on a start-up period or historical data. Therefore, the method is not truly non-parametric.

### 3.4.5 Re-Centring a Process

A further issue in the multivariate case, is what size control adjustment is needed if the process is off-target? It is common for a part with, for example, thirty CtQs to be manufactured in a machining centre that has five axis and therefore only five offsets that can be adjusted. In this case, a single offset adjustment has an effect on multiple CtQ features. This lack of independence between a control feature adjustment and the re-centring of a CtQ, this adjustment procedure is, therefore, a skilled task heavily on operator experience.

How an operator changes control parameters after a off-target signal, is a topic that has had little attention compared to the quantity of research into process control procedures. In the univariate case, the scale and direction of a control parameter change is usually clear. In the multivariate case it is a complex decision, as a change to one control parameter effects multiple design features. However, it is still typical that process operators are left to make this decision.

It is possible to use feedback controllers to derive parameter adjustments. The integration of process control and feedback control has been identified as leading to further practical improvement in quality [22]. This use of discrete process control tools and continuous feedback controllers has been used in various forms and generalised as Run-to-Run (RtR) [117–119]. However, its typical implementation is in batch processes, where the same product is made in continuous batches, such as semiconductor manufacture [120] and chemical industries [121].

These RtR schemes rely on exponentially-weighted moving average SPC charts to identify when a process has changed and no longer on-target. This triggers the use of feedback controllers to prescribe a control action before the next batch of product is started. Although these implementations do not fit with the set-up dominant situation, the philosophy of using process control charts to identify when a process is off-target then feedback controllers to prescribe a control action has potential. Particularly, when a process is multivariate and there are correlations between a single control parameter and multiple design features, using feedback controllers over operator led control actions would increase the consistency of the adjustments made.

### 3.4.6 Summary

This section has shown that there is still a need to develop process control procedures to assist operators of low-volume manufacturing processes. In the univariate case, further development is needed to produce a methodology that can be used consistently across a shop floor on multiple low-volume processes, that can be adjusted to maintain different standards of minimum $C_p$. In the multivariate case, a methodology is required that can monitor all features on one chart, but provides information to the operator that is easy to digestive. Further work is needed, in the multivariate case, to provide a mechanism of feedback that will assist an operator to make changes to control parameters. Chapters 5-7 address these problems.

## 3.5 Summary

In this chapter, process improvement tools that supplement projects using the hybrid Six Sigma process improvement methodology were reviewed. The focus of this review was on tools that allowed completion of the Measure, Analyse and Control stages of the DMAIC cycle. Also included in this review are tools from the Shainin System, that are less well used in traditional Six Sigma, but have great potential to support the application of the DMAIC cycle to low-volume processes. The results of this review identified process improvement tools that are currently state-of-the-art, distinguishing between those that provide an objective approach to analysis and those that provide a subjective approach to analysis, and finally indicating their suitability for application in low-volume processes.

# Chapter 4

# PROcess VAriation Diagnosis Tool (PROVADT)



## 4.1 Introduction

As discrete manufacturing operations move from high-volume to low-volume production runs, there is a tendency to use subjective analysis to resolve quality problems [1]. In this thesis a low-volume production run is treated as one that produces between 5-1,000 units. A new concise process improvement sampling methodology, known as PROVADT, was proposed by Cox et al. [122]. The PROVADT method was designed to increase the objectivity of Six Sigma projects in low-volume manufacture by following the hybrid Six Sigma DMAIC cycle described in chapter 2. This was achieved by sampling from a process, following a set structure that enabled the application of the required statistical metrics from Six Sigmas measure and analyse phases. The PROVADT sampling structure devised by Cox et al. [74, 122], also, allowed the graphical analysis tools from the Shainin System to be used.

PROVADT typically requires the sampling of twenty products each measured three times in a specific sequence. To those sixty results, four improvement tools can be applied: a Multi-Vari chart and an Isoplot from the Shainin system [31, 32, 52]; a Gage R&R analysis and a provisional process capability study from Six Sigma [24, 27, 37]. Compiling the statistical sampling requirements for these techniques into one set of samples and measurements, shortens the analyse phase.

An issue, highlighted in [74, 122], with PROVADT is that it is unable to distinguish between measurement and product variation in the presence of a poor Gage R&R result. In this chapter PROVADT's sampling structure is improved and the issue is addressed by enabling a true Gage R&R as part of its design.

This chapter is structured to provide: an outline of where PROVADT fits in a quality improvement initiative and the tools it implements to achieve this in section 4.2; the sampling constraints required by PROVADT to perform statistically valid analysis are defined in section 4.3; an enhanced approach is suggested in section 4.4 addressing the poor Gage R&R issue of the original PROVADT proposed in Cox et al. [74, 122]; in section 4.5 a worked example is provided which directly compares the original and enhanced approaches; two industrial case studies of PROVADT's implementation are detailed in section 4.6; and conclusions are drawn in section 4.7.

## 4.2 PROVADT Tools

The PROVADT methodology enables a user to move a Six Sigma improvement project from its measure through its analyse phase with a single sampling strategy [74, 122]. This is achieved by utilising principles from the Shainin System. PROVADTs integration of elements and tools from both is shown in Figure 4.1.

PROVADT achieves its analysis by using four process improvement tools: Multi-Vari Chart, Isoplot, Gage R&R and Process Capability Study.

The Multi-Vari chart is the principal technique for clue generation in the Shainin System [32, 51, 52]. Recent examples of its practical implementation, are given in Shanmugam and Kalaichelvan [54], Doganaksoy and Hahn [71], Zaciewski and Németh [72]. The use of Multi-Vari charts within a Six Sigma framework to assist the analyse phase is illustrated in [18, 73]; for recent case studies see [74, 122]. In either context, it allows a practitioner to narrow down to a few potential root causes of variation objectively by means of a stratified experiment which identifies the signature of variation [75, 76].

Isoplots are graphs that are used to identify the scale of variation in a measurement system relative to the variation in the process [32]. This is achieved by two appraisers measuring the same CtQ on typically thirty products. From the results, the measurement system variation is displayed graphically against process variation to see which has the greatest effect.

Gage R&R is a measurement system analysis tool. It is classically used at the measure phase within DMAIC to statistically validate the consistency and stability of the measurement system. $\sigma_E^2$ is the variation caused by the measuring equipment. The ability of a measuring device to provide consistent measurement data is important in the improvement of any process. $\sigma_A^2$ is variation caused by the appraisers, which requires measurement readings to be acquired by different appraisers under the same conditions to be assessed [63]. The

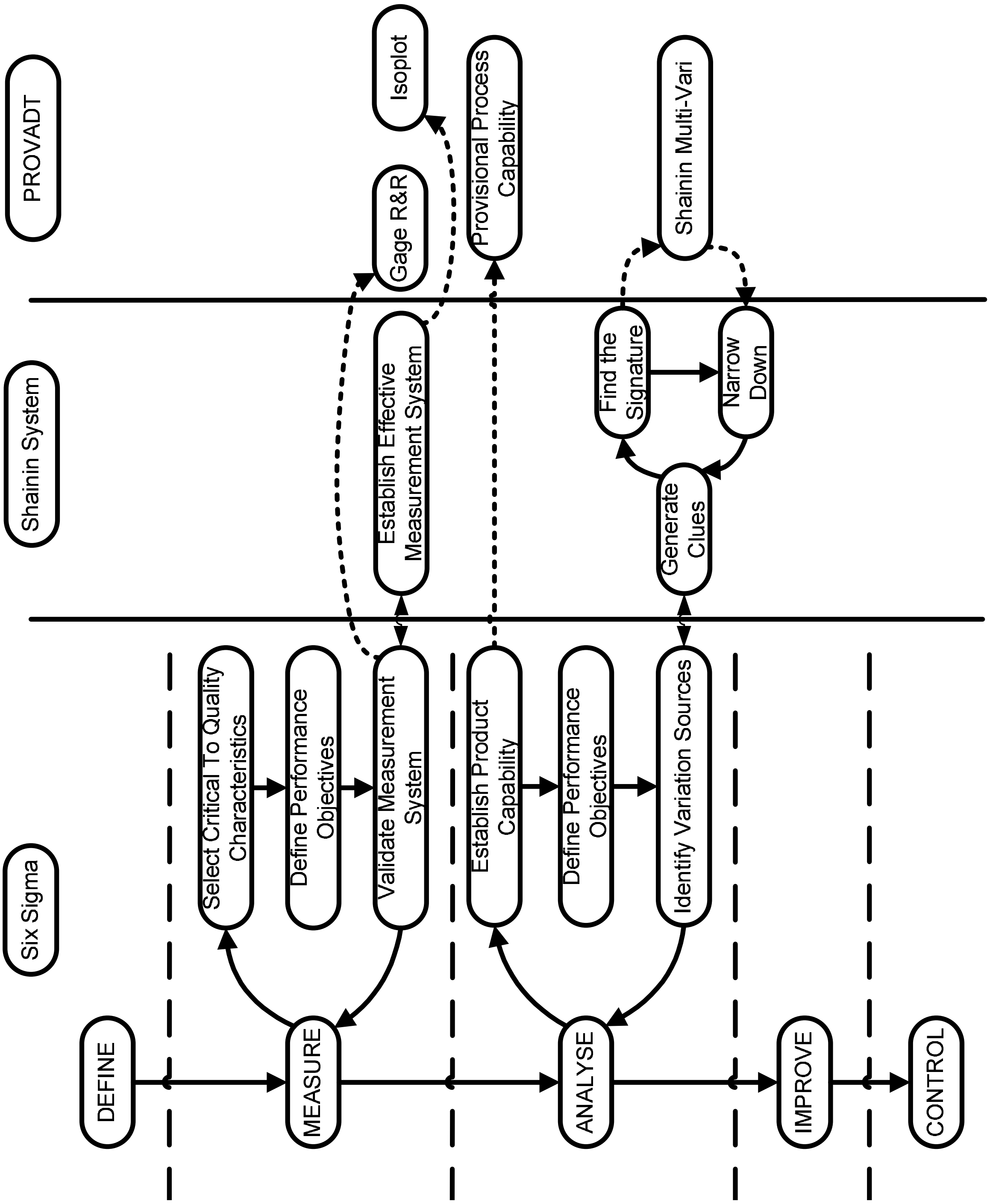


Figure 4.1: Outline of the tools applied by PROVADT.

total Gage R&R variation, which is the sum of $\sigma_E^2$ and $\sigma_A^2$, is then expressed as a percentage of design tolerance. This should be less than 10% to be deemed an adequate measurement system. If it is between 10% and 30% then it is considered marginal and anything above 30% is inadequate [63].

Process capability indices are used to express numerically a processs ability to produce CtQs within their specified tolerances. Two common indices are $C_p$, which represents the process precision, and $C_{pk}$, which reflects the process accuracy and precision. A value of $C_p$ greater than 1.33 identifies most processes as capable of working within the required tolerance.

## 4.3 Sampling Constraints

In order to maintain the statistical significance of the four tools used, PROVADTs sampling strategy must uphold the following strict sampling constraints:

- The number of time periods, $\alpha$, to sample from should be five or greater and there should be at least three number of consecutive samples, $\beta$, in each sample period, to formulate a Multi-Vari study, see section 3.3. Hence, $\alpha \geqslant 5$ and $\beta \geqslant 3$.

- Then $n$ is defined as:

$$n = \alpha\beta, \tag{4.1}$$

  and $n$ must be greater than 10 to produce a provisional process capability, see section 3.3, but ideally at least 20 to produce an initial set of Isoplots. Therefore, $n \geqslant 20$.

- The total number of measurements ($r_{total}$) taken is dependent on three factors: the number of locations to be measured; the number of appraisers to take measurements in each location; and the number of repeat measurements each operator makes in each location. Therefore, $r_i^{(1)}$ expresses the number of repeat measurements taken by appraiser $i$ in location 1. If $\varepsilon_1$ is defined as the total number of appraisers taking measurements in location 1, the total number of measurements taken at location 1 is $\sum_{i=1}^{\varepsilon_1} r_i^{(1)}$. This is then extended to a second measurement location to calculate $r_{total}$, as follows:

$$r_{total} = \sum_{i=1}^{\varepsilon_1} r_i^{(1)} + \sum_{j=1}^{\varepsilon_2} r_j^{(2)}. \tag{4.2}$$

  As a minimum, $\varepsilon_1 \geqslant 2$ appraisers and $\varepsilon_2 \geqslant 1$ appraiser, hence, $r_{total} = \varepsilon_1 + \varepsilon_2$.

- The number of total measurements ($\varphi$) is defined as:

$$\varphi = nr_{total} \tag{4.3}$$

To complete a Gage R&R study [63], $\varphi \geqslant 60$.
From these constraints the minimum sampling requirements for PROVADT are:

- $\alpha = 5$ time periods.
- $\beta = 4$ consecutive parts.
- $n = 20$ total samples.
- $\varepsilon_1 = 2$ and $\varepsilon_2 = 1$, i.e. two appraisers, with $r_1^{(1)} = 1$, $r_2^{(1)} = 1$ and $r_1^{(2)} = 1$ measurements.
- $r_{total} = 3$ repeats.
- $\varphi = 60$ measurements.

The order the appraisers measure in, is swapped after the first 10 products. From this arrangement all four tests (Multi-Vari, Isoplot, Gage R&R and Process Capability) can be performed.

An important issue in PROVADT following the sampling strategy outlined is the possibility of lacking a sufficient Gage R&R, since it only quantifies variation in the measurement system confounded by variation between locations measured within a products CtQ. A Gage R&R which follows industry guidelines, see AIAG [63], only measures a product in one location. This is due to its focus on determining only measurement variation. Hence, the $\sigma_E^2$ in a true Gage R&R is given by:

$$\sigma_E = \frac{6\bar{R_E}}{d_2^*} \tag{4.4}$$

where, $\bar{R_E}$ is where the average range between repeat measurements and $d_2^*$ is Hartleys statistical constant, values can be found in [86]. Hartley's constant is used to approximate a populations standard deviation given the sample size ($n$) of a subgroup. However, PROVADT is a diagnostic tool and therefore aims at capturing all variation sources. This results in a Gage R&R where repeated measurements are taken at different locations on a CtQ. It also confounds measurement and product variation, in which case Equation (4.4) becomes:

$$\sqrt{\sigma_E^2 + \sigma_P^2} = \frac{6\bar{R_P}}{d_2^*} \tag{4.5}$$

where $\sigma_P^2$ is the product-within-piece variation and $\bar{R_P}$ is the average range between repeat measurements at different locations. A low Gage R&R score shows there is little measurement or product variation. However, a high Gage R&R does not clearly identify if this is truly a result of a poor measurement system. This conflicts with traditional Gage R&R assessment approaches and provides the motivation for an enhanced PROVADT method that eliminates this problem.

## 4.4 Enhanced PROVADT

A new method has been developed based on the original PROVADT constraints. Extra constraints must be added to ensure a true Gage RR is obtained, as well as quantifying product-within-piece ($\sigma_P^2$) variation.

In the original PROVADT repeated measurements at the same location would result in a true Gage R&R, but would not capture the effect of $\sigma_P^2$. If repeated readings are taken, in the same experiment, at different locations and by the same appraiser that took repeat readings for the Gage R&R, it is possible to show how $\sigma_P^2$ can be assessed analytically by subtracting Equation (4.4) from Equation (4.5), i.e.:

$$\sigma_P = \sqrt{(\sigma_E^2 + \sigma_P^2) - \sigma_E^2} = \sqrt{(\frac{6\bar{R_P}}{d_2^*})^2 - (\frac{6\bar{R_E}}{d_2^*})^2}. \tag{4.6}$$

Following this approach separates measurement, $\sigma_E^2$, and product-within-piece, $\sigma_P^2$, variation. However, to perform this calculation separate estimates of $\bar{R_E}$ and $\bar{R_P}$ need to be made. Hence, two additional sampling constraints are introduced to capture these estimates:

**A** The minimum repeated measurements in one location are $\sum_{i=1}^{\varepsilon_1} r_i^{(1)} \geqslant 3$ with $\varepsilon_1 \geqslant 2$ and $r_1^{(1)} \geqslant 2$. Repeating measurements by multiple operators in one location is critical to estimate $\bar{R_E}$, enabling the calculation of a true Gage R&R.

**B** Additional measurements on at least $m \geqslant 10$ products to be made by a minimum of one appraiser in two new locations, such that $\varepsilon_2 \geqslant 1$ and $\varepsilon_3 \geqslant 1$. A minimum of three measurement locations is critical to obtaining an estimate of $\bar{R_P}$. Therefore, the total measurements made by enhanced PROVADT, $\varphi_{En}$, are:

$$\varphi_{En} = n\sum_{i=1}^{\varepsilon_1} r_i^{(1)} + m\left(\sum_{j=1}^{\varepsilon_2} r_j^{(2)} + \sum_{k=1}^{\varepsilon_3} r_k^{(3)} + \cdots\right). \tag{4.7}$$

These additional constraints allow for the definition of a minimum sampling strategy for enhanced PROVADT. For example, if products are sampled from a process, $n \geqslant 20$ must be maintained to perform a basic Isoplot. If this sample size is minimised to $n = 20$ products, then this impacts the number of time periods, $\alpha$, to sample from and the number of consecutive products to sample from each time period, $\beta$, such that $\alpha\beta = 20$. To perform a Multi-Vari chart the time periods are minimised to $\alpha = 5$, resulting in $\beta = 4$.

To fulfil the new constraint A for enhanced PROVADT, these 20 products must be measured in one location by a minimum of two appraisers, $\varepsilon_1 = 2$. One of these appraisers needs to repeat these measurements so that $r_1^{(1)} = 2$ and $r_2^{(1)} = 1$, hence, $\sum_{i=1}^{2} r_i^{(1)} = 3$. This set of measurements results in a true Gage R&R, by first calculating $\bar{R_E}$ of appraiser 1 in location 1. To find this, the absolute difference between appraiser 1 measurements in location 1 is averaged:

$$\bar{R_E} = \frac{\sum_{a=1}^{20} \left| x_1^{(1,1,a)} - x_2^{(1,1,a)} \right|}{n}, \tag{4.8}$$

where, $x_i^{(1,1,a)}$ is the measurement taken by appraiser 1 in location 1 for product $a$ of the $n$ products collected. $\bar{R_E}$ is used to estimate repeatability or equipment variation ($\sigma_E^2$) using Equation (4.4) for 20 products and 2 repeats and $d_2^* = 1.128$ [86]:

$$\sigma_E = \frac{6 \times \bar{R_E}}{1.128}. \tag{4.9}$$

In order to calculate an estimate of reproducibility or appraiser variation ($\sigma_A^2$), the mean measurement of repeat 1 for appraiser 1 ($X_1^{\bar{(1,1)}}$) and appraiser 2 ($X_1^{\bar{(2,1)}}$) at location 1 is used as follows:

$$\sigma_A = \sqrt{\left( \frac{6 \times \left| X_1^{\bar{(1,1)}} - X_1^{\bar{(2,1)}} \right|}{d_2^*} \right)^2 - \frac{\sigma_E^2}{nr}}, \tag{4.10}$$

where, $n$ is the number of products and $r$ is the number of repeats. Hence, for minimum enhanced PROVADT, where, $d_2^* = 1.41$:

$$\sigma_A = \sqrt{\left( \frac{6 \times \left| X_1^{\bar{(1,1)}} - X_1^{\bar{(2,1)}} \right|}{1.41} \right)^2 - \frac{\sigma_E^2}{40}}. \tag{4.11}$$

Summing the squares of $\sigma_E$ and $\sigma_A$ enables the calculation of the total Gage R&R ($\sigma_{RR}^{(enhanced)}$) by:

$$\sigma_{RR}^{(enhanced)} = \sqrt{\sigma_E^2 + \sigma_A^2}. \tag{4.12}$$

Also, this set of measurements taken across the 20 products by appraiser one can be used to calculate the capability in this location.

The second new constraint B is fulfilled by re-measuring some of the 20 products by at least appraiser one in at least two new locations. If this constraint is minimised, $m = 10$ products are re-measured in two new locations by one appraiser, hence, $\varepsilon_2 = 1$ and $\varepsilon_3 = 1$. This results in $\sum_{j=1}^{\varepsilon_2} r_j^{(2)} = 1$ and $\sum_{k=1}^{\varepsilon_3} r_k^{(3)} = 1$, therefore, $\varphi_{En} = 80$ total number of measurements in a minimised enhanced PROVADT. It is recommended that these additional measurements taken at new locations are spread across the 5 sample time periods. One way to achieve this is to number sampled products sequentially as they are collected and make

Table 4.1: PROVADT variants, indicating test appraisers and locations for measurements.

| | Appraiser | 1 | 1 | 1 | 1 | 2 |
|---|---|---|---|---|---|---|
| | Location | 1 | 1 | 2 | 3 | 1 |
| Original | All | • | | • | | • |
| Enhanced | Odd Samples | • | • | • | • | • |
| | Even Samples | • | • | | | • |

additional measurements on the odd number samples. From this, within-product variation can be estimated by first calculating $\bar{R_P}$ of appraiser 1 in all locations:

$$\bar{R_P} = \frac{\sum_{b=1}^{10} max\left\{\left|x_1^{1,1,b} - x_1^{1,2,b}\right|, \left|x_1^{1,1,b} - x_1^{1,3,b}\right|, \left|x_1^{1,2,b} - x_1^{1,3,b}\right|\right\}}{10}, \tag{4.13}$$

where, $x_1^{1,y,b}$ is the measurement taken by appraiser 1 in location $y$ for product $b$ of the $m$ products measured in multiple locations. $\bar{R_P}$ is used to estimate the within-product variation ($\sigma_P^2$) using Equation (4.6) for 10 products and 3 repeats and $d_2^* = 1.72$ [86]:

$$\sigma_P = \sqrt{\left(\frac{6R_P}{1.72}\right)^2 - \left(\frac{6R_E}{1.128}\right)^2}. \tag{4.14}$$

A comparison of original and the minimum enhanced PROVADT can be seen in Table 4.1. The use of the enhanced PROVADT procedure to generate Multi-Vari, Isoplot, Gage R&R and Process Capability studies is highlighted in the next section. This provides a test study generated with a manufacturing simulation and is used to provide a worked example.

## 4.5 Simulation Work Example

### 4.5.1 Simulation Set-Up

The results in this section were generated by a teaching simulation known as Process Improvement Methods Simulation (PIMS) [59]. This simulation reflects a discrete batch manufacturing process and allows users to explore different quality improvement methods to determine which stage of the simulated process is the root cause of variation in the products CtQ. Figure 4.2 provides an overview of the PIMS process, which has three manufacturing stages followed by an opportunity to measure the final product.

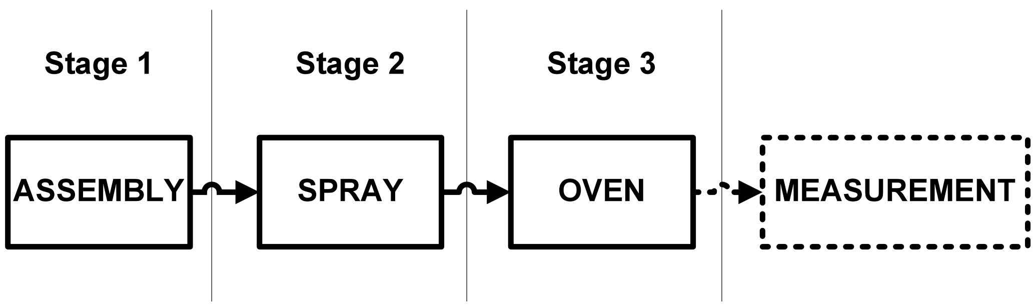


Figure 4.2: Outline of PIMS process.

In the assembly stage five components, A, B, C, D and E, are added onto a substrate. In these components there are different patterns of variation to represent different potential issues. A, B and D contain normally distributed variation; representing slight differences in components of the same type from a supplier. In C there is step-change variation every 18 components to reflect the effect of different batches from a supplier. Component E has variation with-in the component; this is seen as issue of ovality in what should be a perfect circle.

This assembly then goes through a spray process which has variation in two parameters pressure and viscosity, uniformly distributed. The assemblies then go through an oven curing process in batches of nine products, where there is potential variation caused by the oven position occupied during the curing.

The simulation set-up can be modified so that each input variable has some or no effect on the measured CtQ of the final product. The goal of the simulation is to identify which inputs affect the final product allowing the process to be improved. The product has a dimensional CtQ with a unilateral tolerance of target, $T = 100mm$, and lower tolerance limit, $l = 55mm$. To achieve this both a minimised original and enhanced PROVADT where implemented to compare results.

The sampling and measurements for original PROVADT were structured using the minimal setup. To fulfil these requirements, individual batches were selected to represent a single time period. From five separate batches four samples were collected. This gave twenty products to test, and tests were carried out by two appraisers. The results are presented in Appendix A Table A.1.

The sampling and measurements for enhanced PROVADT were, also, structured using the minimal setup. The sampling was conducted in the same manner as the original approach. The results are presented in in Appendix A Table A.2.

These results are then analysed with a Multi-Vari Chart, Isoplot, Gage R&R and Process Capability Study, as follows.

### 4.5.2 Multi-Vari Chart

The Multi-Vari chart uses all gathered measurements which are plotted in Figure 4.3(a) for original PROVADT and in Figure 4.3(b) for enhanced PROVADT. These measurements are divided into three strata on the chart known as: within-piece, piece-to-piece and time-to-time.

In Figure 4.3(a) the largest or Red X variation is within-piece. This is indicated by the vertical groups of three measurements per sample. The second largest or Pink X variation is piece-to-piece and there is little time-to-time effect.

In Figure 4.3(b) the largest or Red X variation is within-piece. This is indicated by the vertical groups of three and five alternating measurements per sample. The second largest or Pink X variation is piece-to-piece and there is little time-to-time effect.

From Figure 4.3(a) and (b), it is important to note that the $\sigma_P^2$ variation is represented

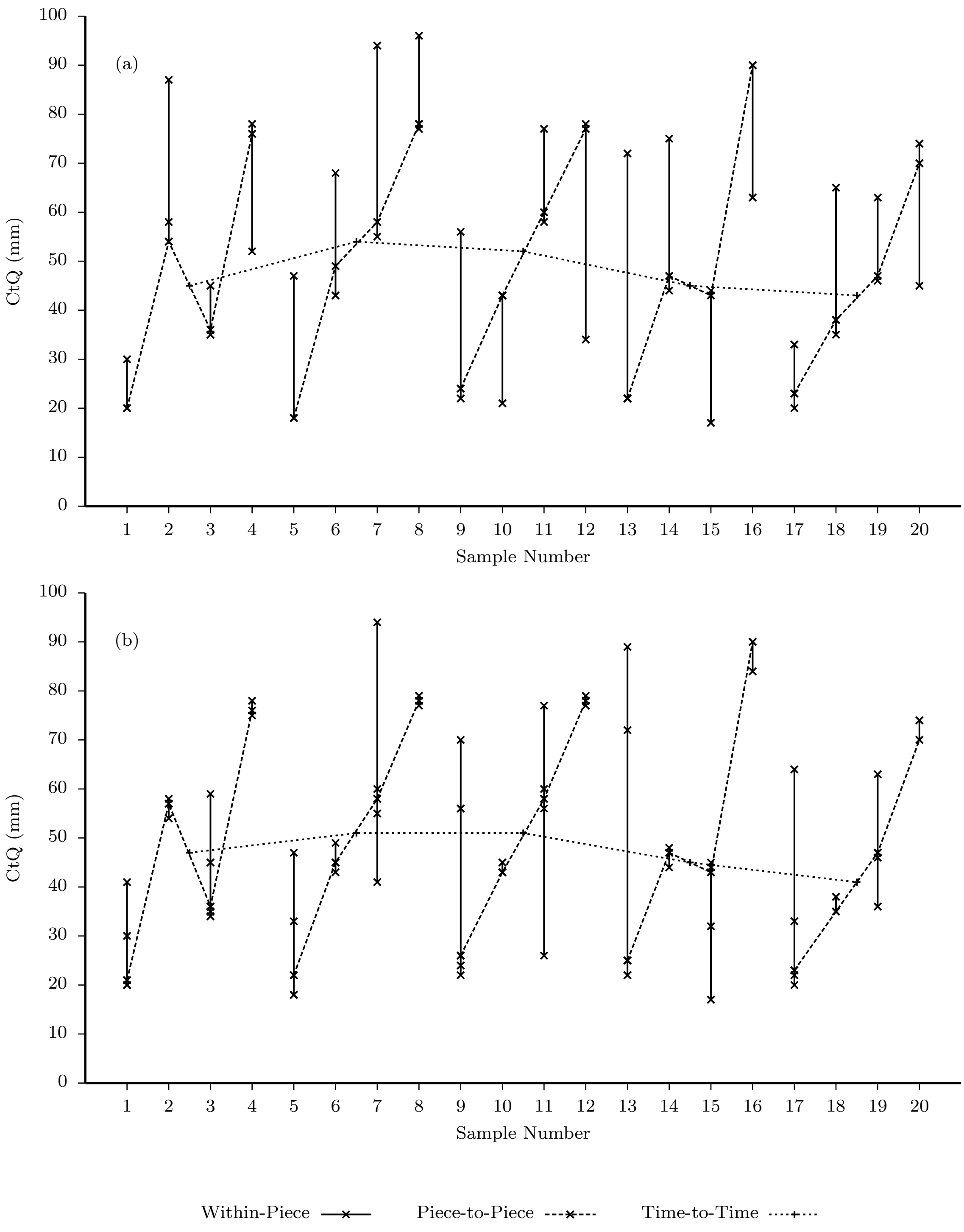


Figure 4.3: Multi-Vari studies of simulation results using (a) original and (b) enhanced PROVADT.

in the within-piece variation on all products using original PROVADT, but only represented in odd numbered products using enhanced PROVADT. However, both approaches identify within-piece variation as the Red X. Following the Shainin System philosophy the root cause of the Red X variation is the first to be investigated. This was done initially by examining the results again using an Isoplot study in the next section.

### 4.5.3 Isoplots

From the data in Appendix A Table A.1 and A.2, two Isoplots were created. The first plot compares the measurement repeatability of appraiser 1. To do this, the two repeated measures taken in the same location were plotted against each other in Figure 4.4(a) and (c) for original and enhanced PROVADT, respectively. The second Isoplot compares the measurement reproducibility between appraisers 1 and 2. This variation is shown in Figure 4.4(b) and (d) for original and enhanced, respectively, by plotting the results of appraiser 1 against appraiser 2 from the same location.

The plots in Figure 4.4 show the results as one homogeneous group, indicating that the test does not affect the process. Highlighted in Figure 4.4(a), is that there is variation both along the 45° line and perpendicular to it, suggesting that there is excessive repeatability variation. This indicates that the measurement system is contributing to the within-piece Red X. However, this result should be taken with caution since as mentioned before original PROVADT confounds repeatability and product-within-piece variation. Figure 4.4(b) shows that there is more variation along the 45° line than perpendicular to it. This indicates little reproducibility variation in the measurement system. Both Figure 4.4(c) and (d), also highlight, that there is more variation along the 45° line than perpendicular to it. This means that there is greater piece-to-piece process variation than measurement variation, indicating that the measurement system is not contributing to the within-piece Red X. This finding is quantified by a Gage R&R study as shown in the next section.

### 4.5.4 Gage R&R Study

A derivation of a Gage R&R study for original PROVADT is in Cox et al. [74]. This was applied to the sampling results at Appendix A Table A.1 to give a total Gage R&R, as a percentage of tolerance, $\sigma_{RR}^{(original)} = 308.9\%$. This suggests that the measurement system is inadequate, due to $\sigma_{RR}^{(original)} \geq 30\%$ [63]. In Figure 4.5 it is shown that the repeatability variation is the biggest contribution to the overall Gage R&R effect. This is consistent with the Isoplot observations in Figure 4.4(a) and (b).

The advantage of the enhanced PROVADT method is that a true Gage R&R value is calculated. In the case of the simulation, this is achieved by using the sampling results in Appendix B and Equations (4.4) - (4.14), which gives $\%\sigma_{RR}^{(enhanced)} = 22\%$. Using the guidelines from the TS/ISO 16949 standard reference manual, the score of 22% would rate this as a marginal measurement system.

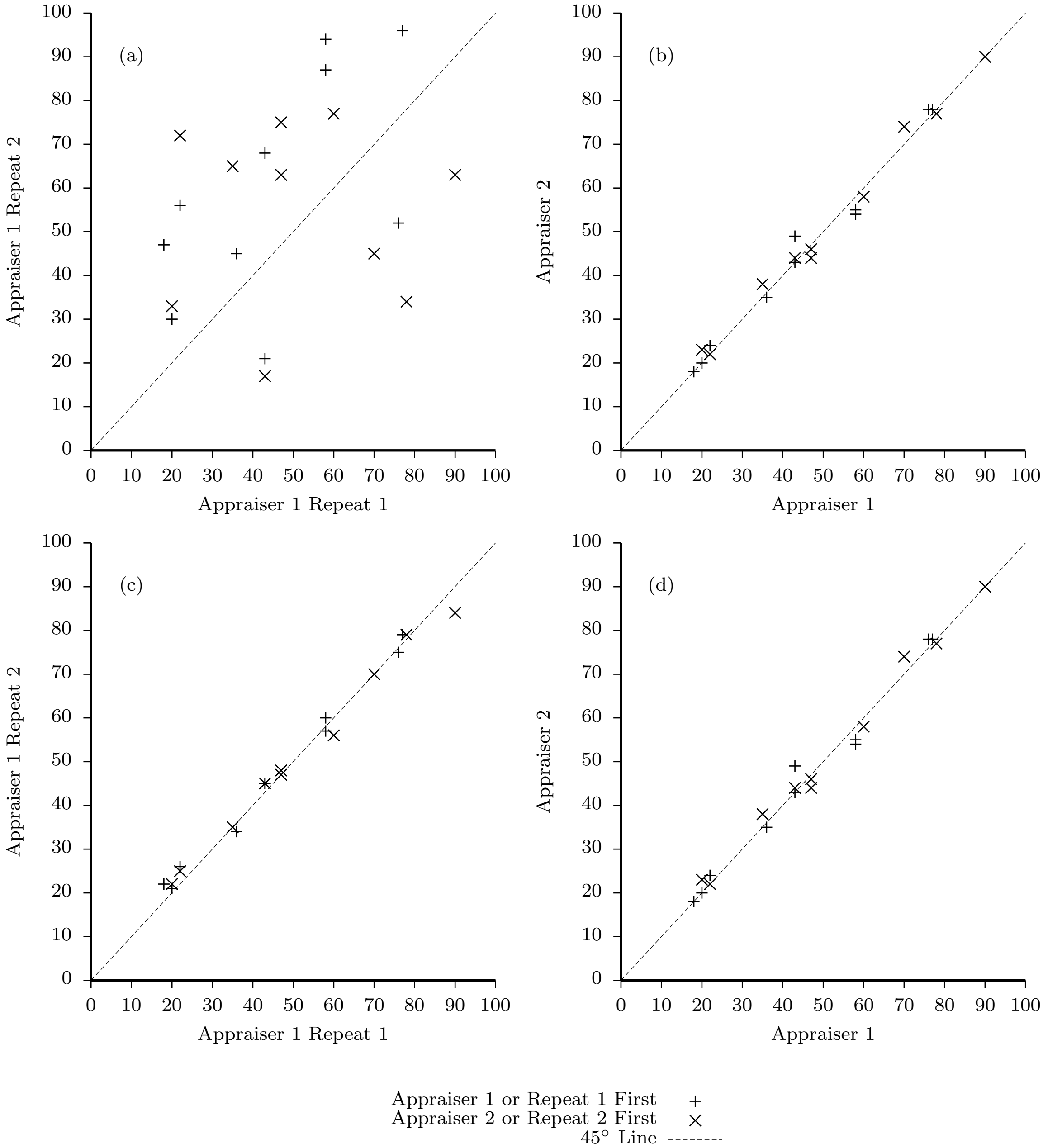


Figure 4.4: Isoplots of simulation original results in: (a) Repeatability of appraiser 1 and (b) Reproducibility of appraiser 1 against appraiser 2; enhanced results in: (c) Repeatability of appraiser 1 and (d) Reproducibility of appraiser 1 against appraiser 2.

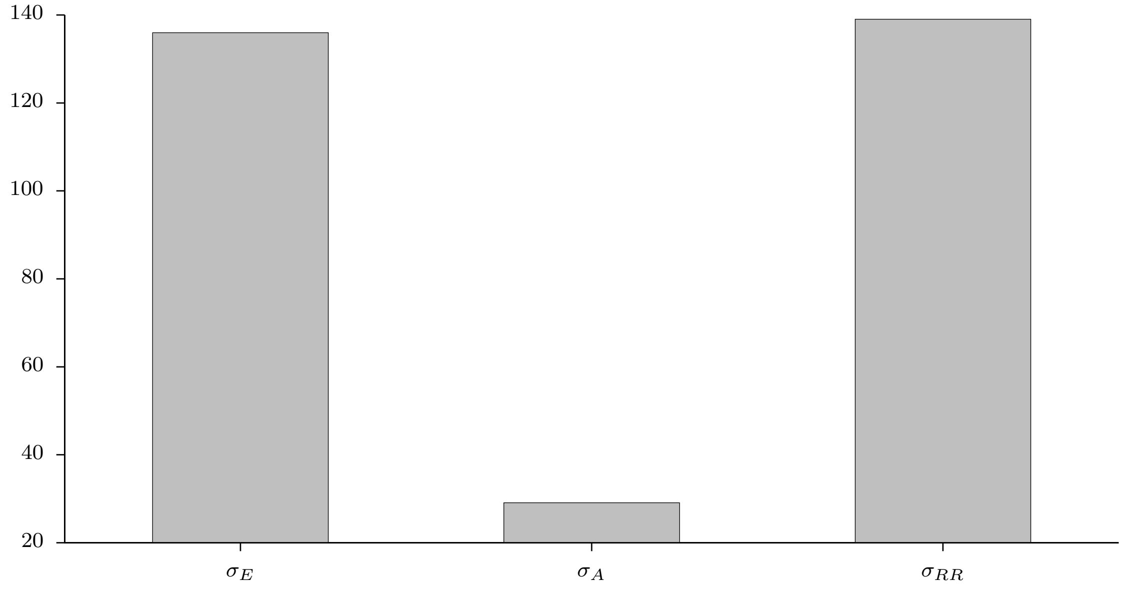


Figure 4.5: Gage R&R of simulation results analysed with original PROVADT

The individual components of variation are summarised in Figure 4.6, where the effect of variation within the product itself is considerably larger than the measurement system variation. This is consistent with the findings from the Isoplots in Figure 4.4(c) and (d). It also further narrows down the potential root causes of the within-piece Red X variation as being a result of the product and not the measurement system.

This set of results shows a significant difference in the original and enhanced PROVADT methods in determining a true Gage R&R. At this stage the original approach would have determined that the within-piece Red X variation, identified in the Multi-Vari study, was a result of an inadequate measurement system. However, due to the known confounding of repeatability and product-within-piece variation, a follow-up gage analysis would need to be performed to separate these issues. This would take more time and testing resource, as a fresh gage analysis uses a minimum of 60 new measurements. In the case of the enhanced approach, no further gage studies are required, with the addition of only 20 measurements to the original PROVADT. Not only, are the repeatability and product-within-piece variations not confounded, but all variation sources are quantified. Thus, the within-piece Red X is further narrowed down to an issue of non-uniformity in the product. This clearly shows the improvement of the enhanced PROVADT over the original.

### 4.5.5 Provisional Process Capability Study

Just as the Gage R&R quantifies the within-piece variation seen in the Multi-Vari chart, the provisional process capability study quantifies the piece-to-piece variation. The enhanced PROVADT allows the calculation of process capability at the three measured locations.

In this worked example the product has a unilateral tolerance; therefore, only $C_{pk}$ is calculated. For original PROVADT this is performed for the two measurement locations by

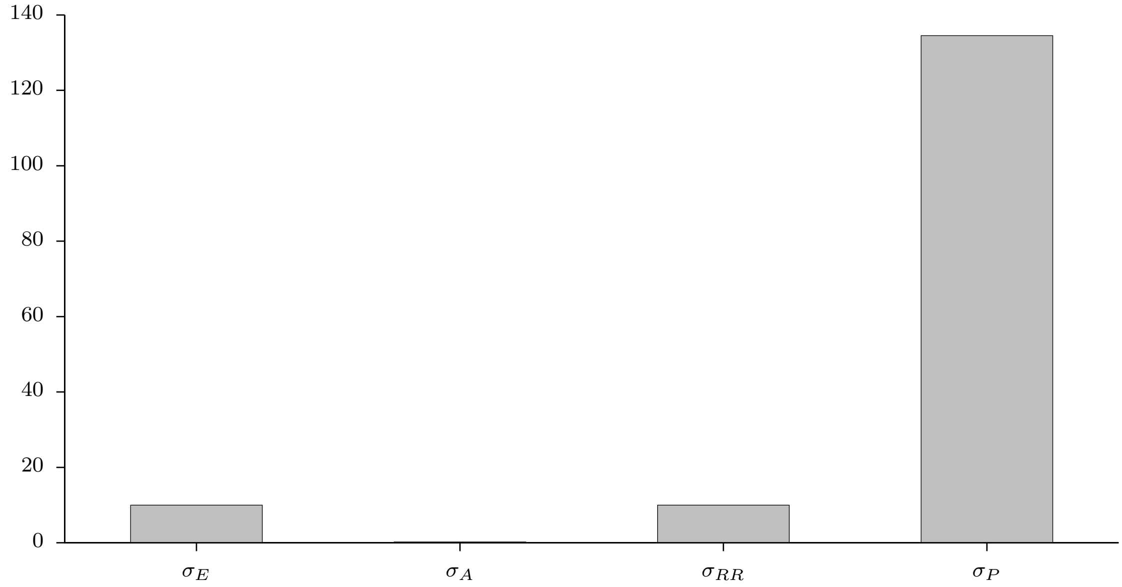


Figure 4.6: Gage R&R of simulation results analysed with enhanced PROVADT including within-piece variation sources.

using average results for appraiser 1 across the 20 products i.e. $\bar{X}_1^{(1,1)}$, $\bar{X}_1^{(1,2)}$ and Equation (4.2). This leads to $C_{pk}^{(1)} = -0.1023$ and $C_{pk}^{(2)} = -0.0289$, which implies the process has a poor overall capability. These results are plotted in Figure 4.7(a). Given the inadequate Gage R&R evaluation these results would be considered not reflective of the process but of the excessive measurement variation. Again, this highlights the importance of obtaining a true Gage R&R.

For enhanced PROVADT, the capability calculations are performed using the location means for appraiser 1 repeat 1 ($\bar{X}_1^{(1,1)}$, $\bar{X}_1^{(1,2)}$,$\bar{X}_1^{(1,3)}$) and Equation (4.2). This leads to $C_{pk}^{(1)} = -0.1023$, $C_{pk}^{(2)} = -0.0236$ and $C_{pk}^{(3)} = -0.0981$, which implies the process has poor capability in all three locations. The negative results indicate that the process mean is outside of the tolerance limit. This capability study is consistent with the Multi-Vari chart in that, despite the Red X within-piece issue, there is a piece-to-piece Pink X underneath, that is quantified by a poor $C_{pk}$ values. These results are summarised in Figure 4.7(b).

It is clear to see that both original and enhanced PROVADT offer similar capability results. The two advantages the enhanced method has shown is that it provided the capability in a third location and the complement of a valid Gage R&R result allows these results to be used without further testing.

### 4.5.6 Summary

Both original and enhanced PROVADT assisted the user to narrow down towards the root cause of variation. In this example it was an ovality issue in the circle component E that caused product-within-piece variation in the final CtQ. This was determined by analysing product using a technique such as paired comparisons, with a reduced number of potential

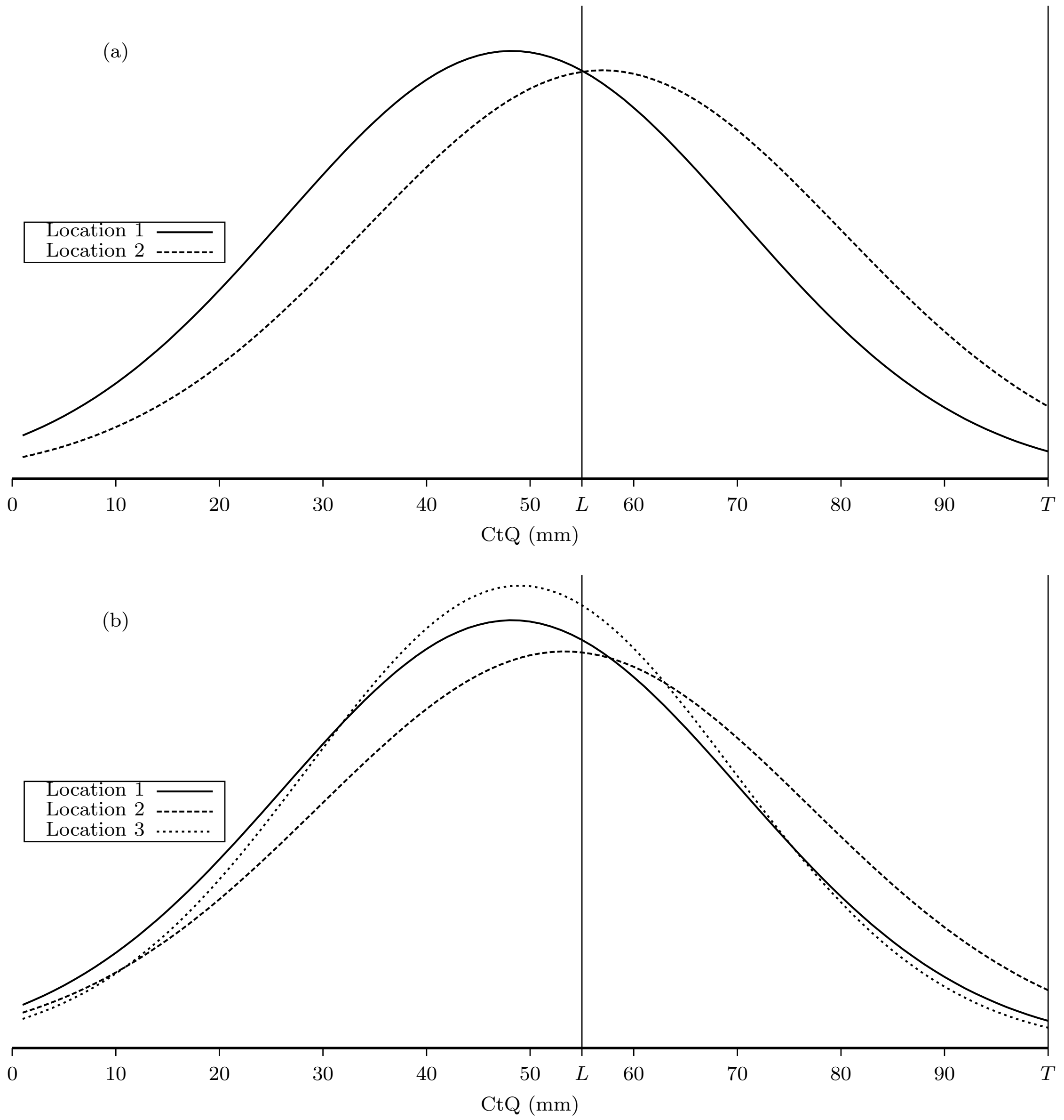


Figure 4.7: Provisional process capability of simulation results from (a) original and (b) enhanced PROVADT.

variation sources. However, the difference between the original and enhanced approaches was that the original approach needed further experimentation to validate the measurement system. Whereas, enhanced could go straight into the paired comparison analysis. The following section details two industrial case studies are presented were enhanced PROVADT has been applied.

## 4.6 Industrial Case Studies

### 4.6.1 Overview

In this section two industrial case studies are detailed, where enhanced PROVADT has been applied. The first is at Rettig, Team Valley, UK, who manufacture steel radiators for the Myson and Purmo brands. The second is at Coveris, Stanley, UK, who manufacture rigid packaging for the food industry. In both cases samples are collected as per the minimum enhanced PROVADT structure. Then these results are analysed, in the same manner as previously outlined, by Multi-Vari chart, Isoplot, Gage R&R study and Provisional Process Capability, in that order. The subsequent actions in these cases are then summarised.

### 4.6.2 Rettig

#### 4.6.2.1 Background

Rettig brings together Europe's leading brands for radiators, underfloor heating, valves and controls. Its site at Team Valley was built in 1958, and was acquired by Rettig (Myson) in 2000, to manufacture radiators for the Myson and Purmo brands. In 2003 Rettig invested 30 million euros in a new production hall, equipped with two new fully-integrated high-speed welding lines. Down these two lines, called Round Top 1 (RT1) and Round Top 2 (RT2), the side panels for all the non-electric radiators are manufactured. The only difference between the two lines is RT1 is measured in metric units; whereas RT2 is measured using imperial units. The two lines work as a mirror image along the factory.

Both assembly lines produce single and double panelled radiators. A welding wheel running down the edge of each panel creates the top seam. The double panelled radiators are two single panels manufactured one after each other in which the first panel is rotated and then lifted to enable the second panel to move underneath for welding. Once the radiators have been through testing and painting an optional grilling attachment is placed on top.

#### 4.6.2.2 Problem Definition

The problem Rettig experienced was the positioning of grilling on double panelled radiators. The top seams on each panel were not parallel along their own length and, hence, the grille on occasion did not fit flush on top of the radiator. Rettig wanted to understand the root cause of this variance and to compare the scale of variation in both the RT1 and RT2 production lines. It was decided that both the seam depth and height of each panel must be measured to capture all potential sources of variation. This means that four concurrent enhanced PROVADT studies took place.

On both production lines every radiator is manufactured to a technical specification. The panel height and seam depth is specified with a $6mm$ tolerance ($-2mm + 4mm$ either side of the target). Each panel was manufactured to the same specification. This meant that all the radiators coming off both production lines had the potential to be used for a double panelled radiator with grilling. They should have had a parallel top seam so that all radiators could be tested.

#### 4.6.2.3 Measurement Method

The testing at Rettig is non-destructive and data were gathered as per the minimum enhanced PROVADT requirements. This meant that four consecutive products were collected at five time periods resulting in a total of twenty samples. Once the samples were collected, five measurements were taken from every other of the twenty products (odd numbered sample) and thee measurements taken from the remainder (even numbered sample). Appraiser 1 measured twice in a primary location for all samples and Appraiser 2 measured once in the same primary location. Appraiser 1 then measured twice more in a secondary and tertiary location for each odd numbered sample. The order in which the appraisers measured was swapped after 10 products had been measured.

The seam depth was measured with the depth gauge on Vernier callipers and a larger version was used to measure the radiator height. Both sets of callipers had a precision of $0.01mm$. There was no other measuring instrument available; hence, the two appraisers for this testing were two different operators. The results for RT1 and RT2 panel heights and seam depths are presented in Appendix B.

#### 4.6.2.4 Multi-Vari Chart

Multi-Vari charts were derived for the four data sets, seam depth and panel height for both RT1 and RT2,. These show, that in all cases, the Red X is Within-Piece but with a Pink X of Piece-to-Piece. These Multi-Vari charts are shown in Figure 4.8(a)-(d).

The difference between the within-piece points show the variation caused by the product and measurement system. The larger within-piece spread on odd samples indicates that the product variation has a large contribution to the overall variation.

#### 4.6.2.5 Isoplots

In this analysis, the within-piece measurement variation is compared with the piece-to-piece variation, this indicates whether the measurement system is capable relative to the overall piece-to-piece variation. The results for RT 1 and RT2 radiator heights and seam depths are shown in Figure 4.9(a)-(d) and Figure 4.10(a)-(d).

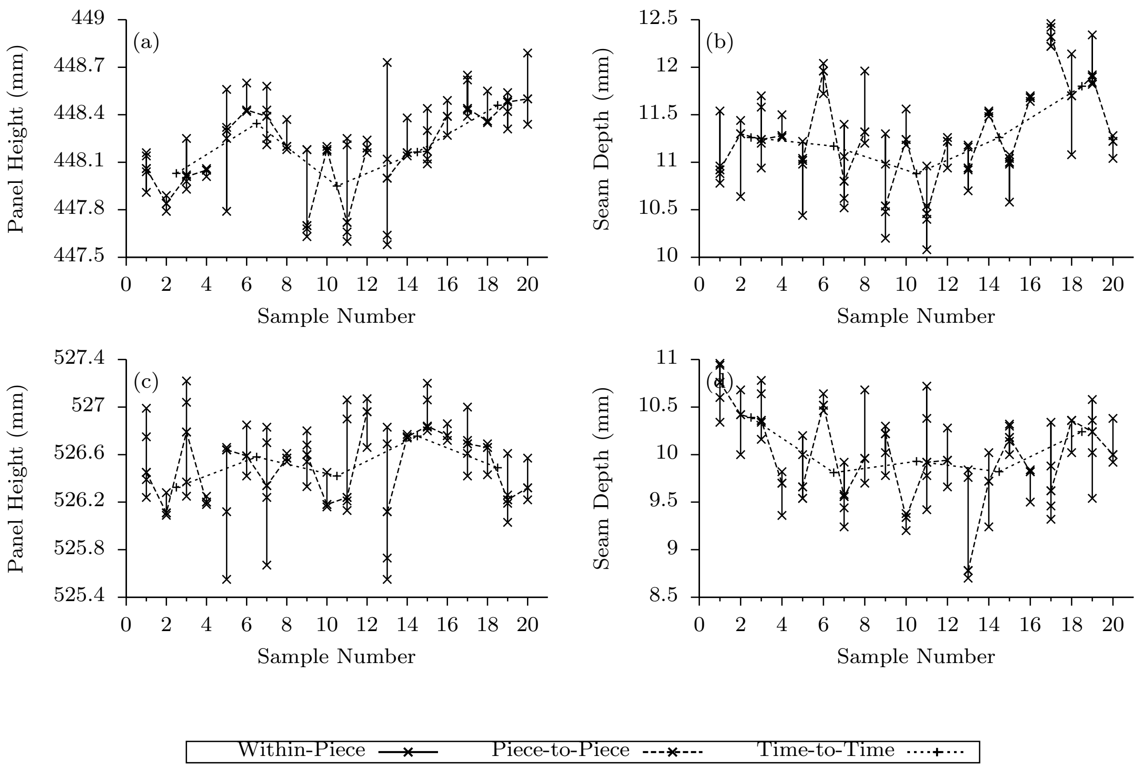


Figure 4.8: Multi-Vari charts for (a) RT1 panel height; (b) RT1 seam depth; (c) RT2 panel height; (d) RT2 seam depth.

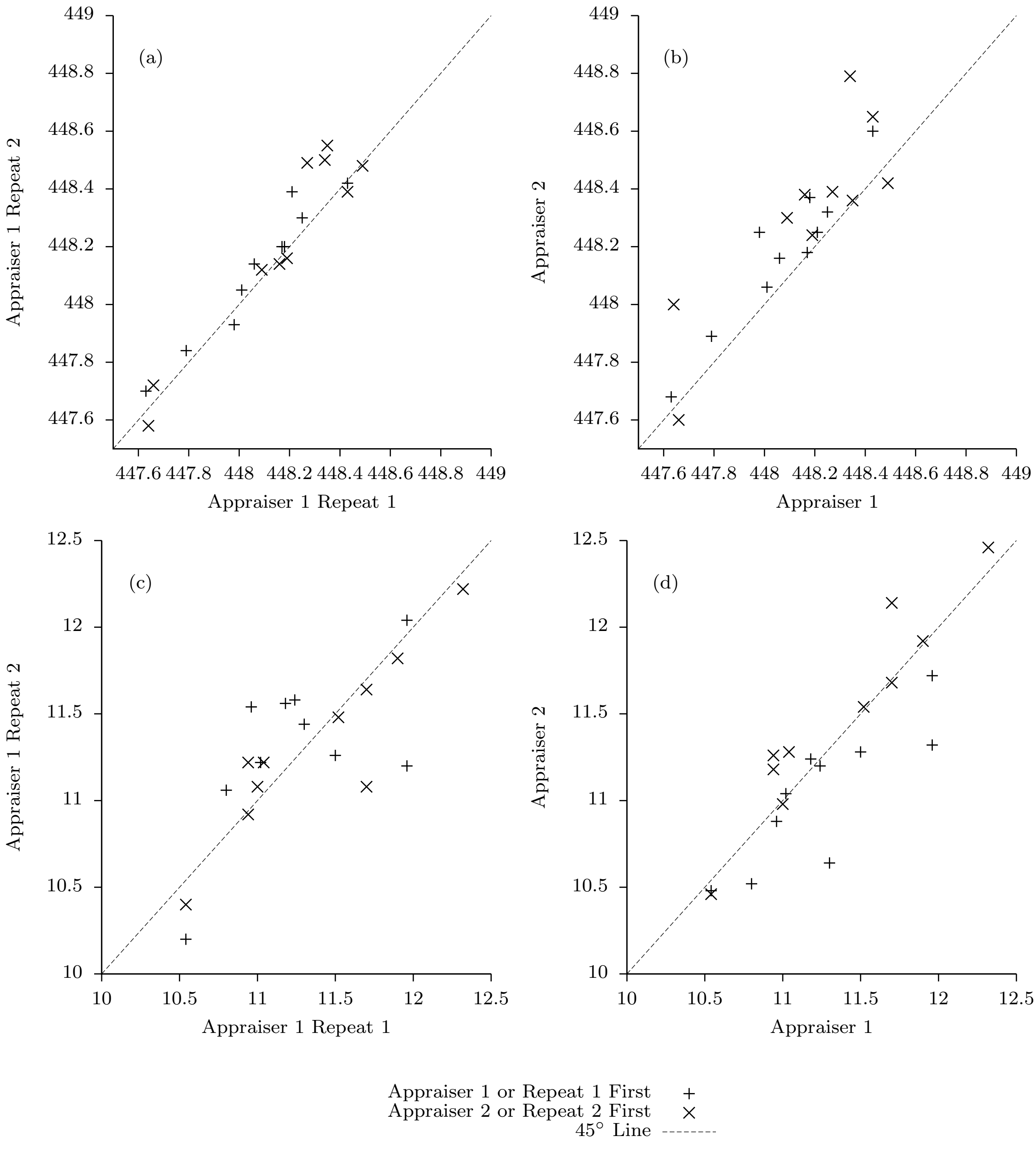


Figure 4.9: Isoplots for RT1 panel height between (a) appraiser 1 repeat measures and (b) appraisers; RT1 seam depth between (c) appraiser 1 repeat measures and (d) appraisers.

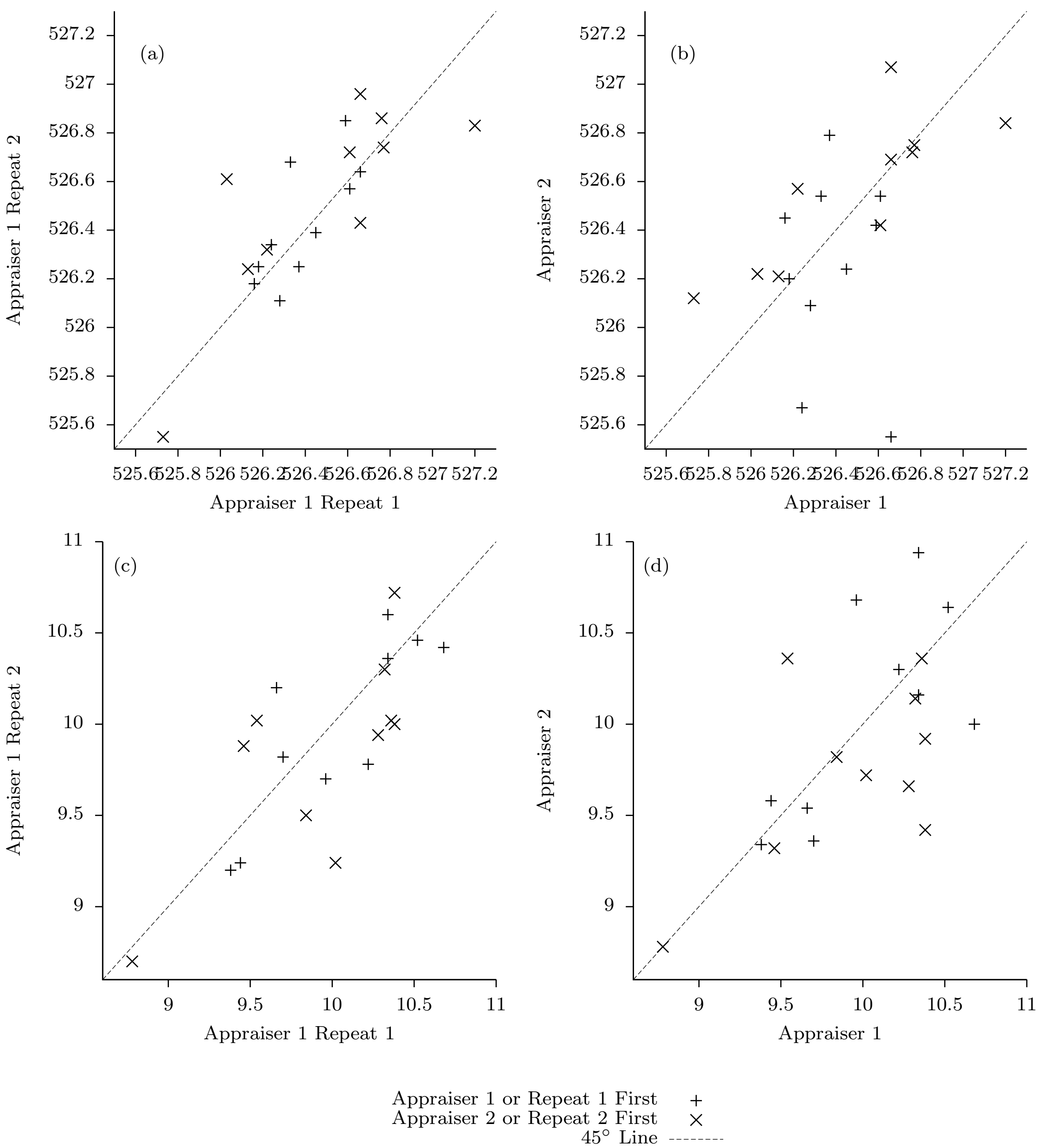


Figure 4.10: Isoplots for RT2 panel height between (a) appraiser 1 repeat measures and (b) appraisers; RT2 seam depth between (c) appraiser 1 repeat measures and (d) appraisers.

Table 4.2: Gage R&R analysis for seam depth and radiator height on RT2 and RT1.

| | RT2 | | RT1 | |
|---|---|---|---|---|
| | Seam Depth (mm) | Rad. Height (mm) | Seam Depth (mm) | Rad. Height (mm) |
| $\sigma_P$ | 0.799 | 1.959 | 1.525 | 1.409 |
| $\sigma_E$ | 1.559 | 0.883 | 1.309 | 0.375 |
| $\sigma_A$ | 0 | 0.268 | 0 | 0.333 |
| $\sigma_{RR}$ | 1.559 | 0.923 | 1.309 | 0.502 |
| $\%\sigma_{RR}$ | 25.9% | 15.3% | 21.8% | 8.3% |

In all charts there are no obvious separate groupings, indicating that the measurement system is not having an effect on the product. Figure 4.9(a) and (c) and Figure 4.10(a) and (c) show the Isoplot for repeatability or consistency of the measurement system between repeated measurements by the same appraiser in the same location. Results are spread along the 45° line than away from it, highlighting that there is more piece-to-piece than repeatability variation in the measurement system. Figure 4.9(b) and (d) and Figure 4.10(b) and (d) show the reproducibility or variation between different appraisers measurements in the same location. These demonstrate more piece-to-piece variation than measurement system variation due to different appraisers. However, there is clearly a large deviation in some results away from the 45° line; therefore, reproducibility is the most significant component of measurement variation.

#### 4.6.2.6 Gage R&R

From the Multi-Vari it was shown that there is within-piece variation caused in part by the product. The Isoplot verified that the measurement system is a non-destructive test. Therefore, calculation of a Gage R&R value is imperative to quantify statistically how much of the within-piece variation is due to measurement. This was unachievable with the previous version of PROVADT; however, it is possible with enhanced PROVADT and the results are displayed in Table 4.2. In this table, $\sigma_P$, $\sigma_E$, $\sigma_A$ and $\sigma_{RR}$ have been calculated using Equations (4.8)-(4.14).

Table 4.2 highlights that the measurements systems should be categorized as marginal, according to AIAG [63], meaning they are sufficient for non-critical CtQs. The results in Table 4.2 also show the relative differences between $\sigma_P$, $\sigma_E$ and $\sigma_A$. This highlighted that the product-within-piece, $\sigma_P^2$, component of variation is at least twice as large as the total measurement-within-piece variation, $\sigma_{RR}^2$, for RT1 and RT2 radiator heights. Therefore, to reduce the size of the Red X within-piece variation, the investigation focused on causes of $\sigma_P^2$ that result in a non-uniform radiator height in the product.

#### 4.6.2.7 Process Capability

A final analysis from the enhanced PROVADT sampling allows for the calculation of a provisional process capability study in each measurement location in order to quantify statistically

Table 4.3: Provisional process capability results for RT2 and RT1.

| | RT2 | | | | | | RT1 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Seam Depth | | | Rad. Height | | | Seam Depth | | | Rad. Height | | |
| Location | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| $C_p$ | 2.04 | 2.39 | 2.28 | 3.03 | 3.36 | 3.36 | 2.00 | 1.25 | 2.07 | 3.80 | 4.74 | 3.93 |

the Pink X piece-to-piece variation. The results of this study are presented in Table 4.3, where it is shown that there is consistency in capability across all measurement locations, since the variation in each location is of a similar magnitude. This is a result of consistent variation across the product and not a result of one location being constantly different from the others. Given that this is a provisional study with small sample sizes, larger discrepancies would be needed to alert an operator.

The provisional process capability results for RT1 and RT2 panel height and seam depth, in Table 4.3, show that these processes are very capable. The minimum $C_p$ of 1.25 demonstrates that the piece-to-piece variation is small relative to the design tolerances.

However, the $C_{pk}$ values for panel height on both RT1 and RT2 were lower than the $C_p$. This indicated that the process was not operating exactly on the design target. This was quickly rectified and the seam welding machines were readjusted to move the process closer to the technical target. Given the high $C_p$ values once the process had been moved back on target there was no economic reason to minimise the Pink X variation further.

#### 4.6.2.8 Overall Analysis Provided to Rettig

Enhanced PROVADT enabled the analysis of production lines RT2 and RT1 from one set of 20 products. It was shown that the main source of variation was within-piece. The majority of this variation was due to non-uniformity in the product, which is now under further investigation.

As a result of the experimental work a small adjustment was made to the measurement system. This improved on the height measurement between RT2 and RT1, resulting in a decrease in the Gage R&R value so the measurement system became adequate.

Process Capability was above 1.33 on both production lines with regards to the seam depth, so it is within its specified tolerance and once the welding machines were realigned the panel height also operated with a high capability. The data collected confirmed Rettigs suspicions that although the process has the capability to produce within the specification limit it can have within-piece variation, within the top seam.

### 4.6.3 Coveris

#### 4.6.3.1 Background

Coveris is a company formed from the merger of five top-ranking plastics packaging companies. Currently, Coveris is the $6^{th}$ largest plastics packaging company in the world. Its site

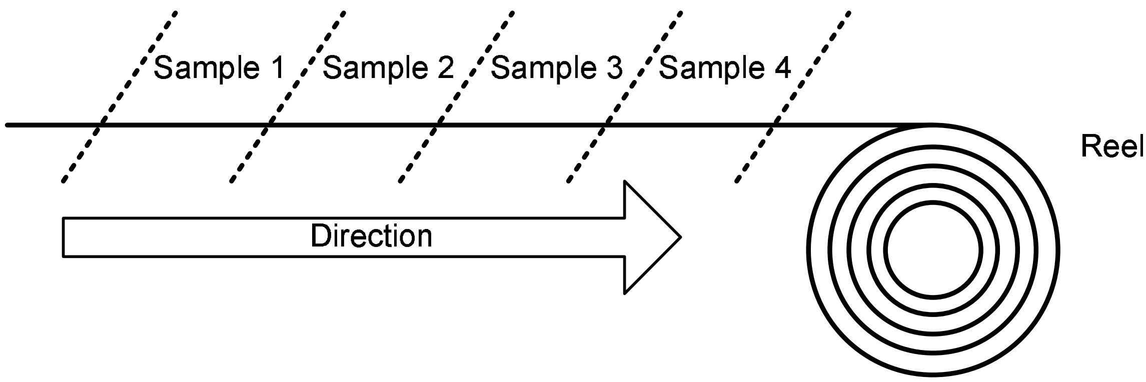


Figure 4.11: Sample collection points from a single reel.

in Stanley, UK, specialises in the co-extrusion and thermoforming of rigid packaging for the food industry.

#### 4.6.3.2 Problem Definition

Coveris recently installed a new co-extrusion facility at its Stanley site, known as Erema. This facility processes raw plastic into rolled sheets which are then used for the thermoforming of food packaging. The Erema process produces batches of sheet plastic, which are wound up into reels. A typical production run produces 5-10 reels. The machine set-up is then changed to produce a product with a different chemical composition and/or gauge of thickness.

As this process was newly established, the company wanted to investigate the capability and variation patterns of plastic gauge thickness in the reels. As the process contains over 15 controllable parameters, Coveris wanted to narrow its focus down to those parameters that account for the most variation and that needed adjustment. To achieve this, enhanced PROVADT was implemented.

#### 4.6.3.3 Measurement Method

In order to gather samples as required by enhanced PROVADT, five consecutive batches (reels) from the same production run where selected to represent time-to-time variation. From these reels a section of plastic sheet was cut away from the end, which was then subdivided into four consecutive samples, as shown in Figure 4.11.

To measure the plastic gauge thickness, a custom depth gauge jig was used and two operators were used as appraisers. Each sample was then measured in a primary location twice by appraiser 1 and once by appraiser 2. This primary location is labelled as (a) in Figure 4.12. Appraiser 1 measured in this location first for the first half of the samples. Appraiser 2 then measured first for the second half of the samples.

Appraiser 1 then measured every alternate or odd numbered sample in a secondary and tertiary location, which are marked as (b) and (c), respectively, in Figure 4.12. These extra

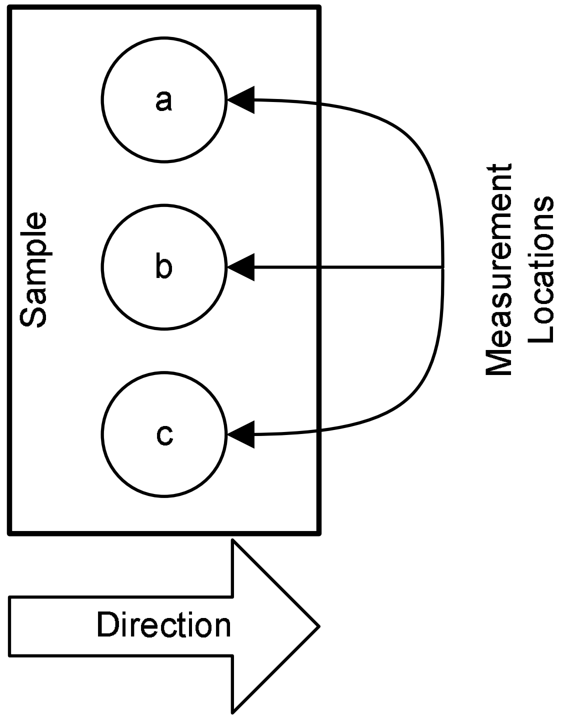


Figure 4.12: Measurement locations within a single sample.

location measurements provide information assessing product-within-piece variation. The target gauge for these samples was $1.150 \pm 0.058mm$ and the recorded measurements are presented in Appendix C.

#### 4.6.3.4 Multi-Vari Chart

Figure 4.13 shows the Multi-Vari chart plotted for the plastic gauge thickness results. It is clearly shown that the Red X is within-piece variation, and a piece-to-piece Pink X.

Therefore, this project focused on understanding the cause of this variation, which could be measurement or product-within-piece variation. Isoplots were used for narrowing down the root cause.

#### 4.6.3.5 Isoplots

The repeatability and reproducibility Isoplots generated from the enhanced PROVADT sampling are displayed in Figure 4.14.

In both Figure 4.14(a) and (b) there are no obvious separate groupings, hence, the measurement system is not effecting the product. It is also shown that the spread of results is greater along the 45° line than across it. This indicates that the piece-to-piece variation is greater than the measurement-within-piece variation, implying that the Red X is a result of product-within-piece variation. These implications are further quantified by the Gage R&R study in the following section.

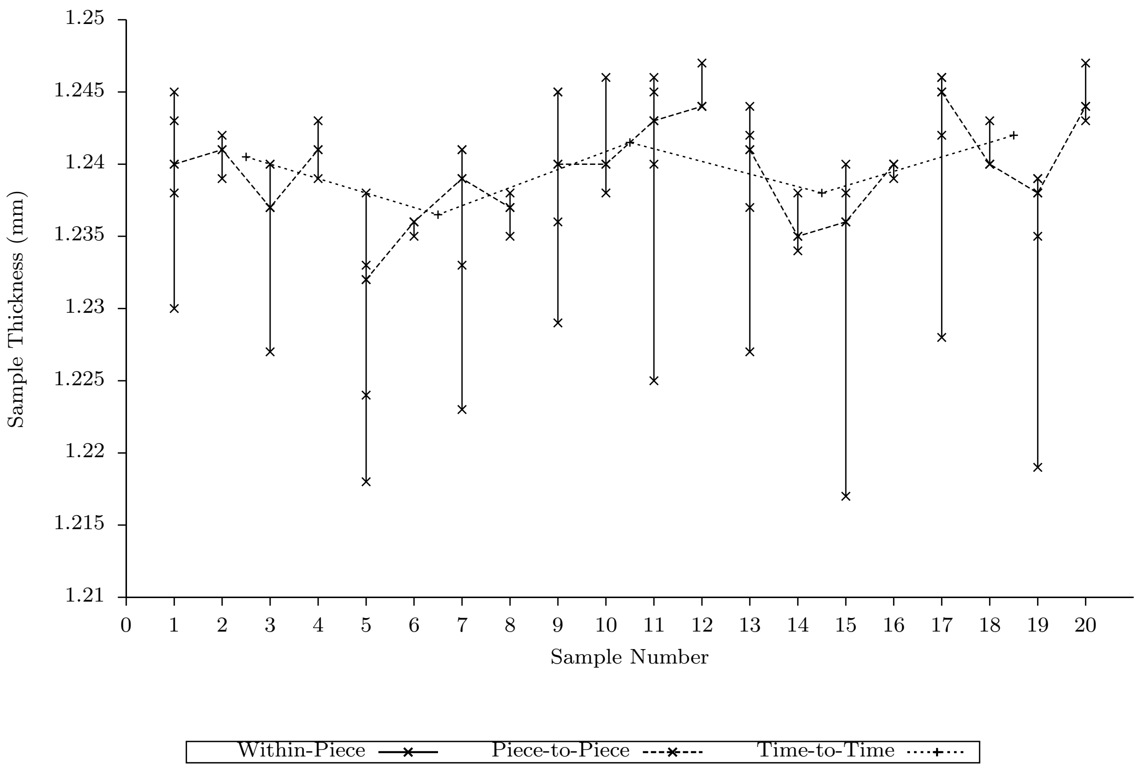


Figure 4.13: Multi-Vari chart for sample thickness.

Table 4.4: Gage R&R for sample thickness.

| | Thickness (mm) |
|---|---|
| $\sigma_P$ | 0.0596 |
| $\sigma_E$ | 0.0114 |
| $\sigma_A$ | 0.0090 |
| $\sigma_{RR}$ | 0.0145 |
| $\%\sigma_{RR}$ | 12.64% |

#### 4.6.3.6 Gage R&R

The Gage R&R results are given in Table 4.4, which were calculated based on the measurements recorded in Appendix C. The two key results from this analysis are the percentage Gage R&R score and the relative size of the $\sigma_P$ against this measurement deviation.

The Gage R&R of 12.64% categorises the measurement system as marginal but close to the adequate boundary. This gave the investigation confidence that the measurement system was able to detect changes in the process that were significant relative to the tolerance. It also reinforced the finding, from the Isoplot, that the Red X is unlikely to be a result of measurement variation. The second analysis, comparing $\sigma_P$ and $\sigma_{RR}$, can be done by reviewing the variance values presented in Table 4.4, more clearly displayed in Figure 4.15.

The bar chart in Figure 4.15 graphically shows that the $\sigma_P$ is four times larger than $\sigma_{RR}$. This confirms that the Red X within-piece variance is a result of product-within-

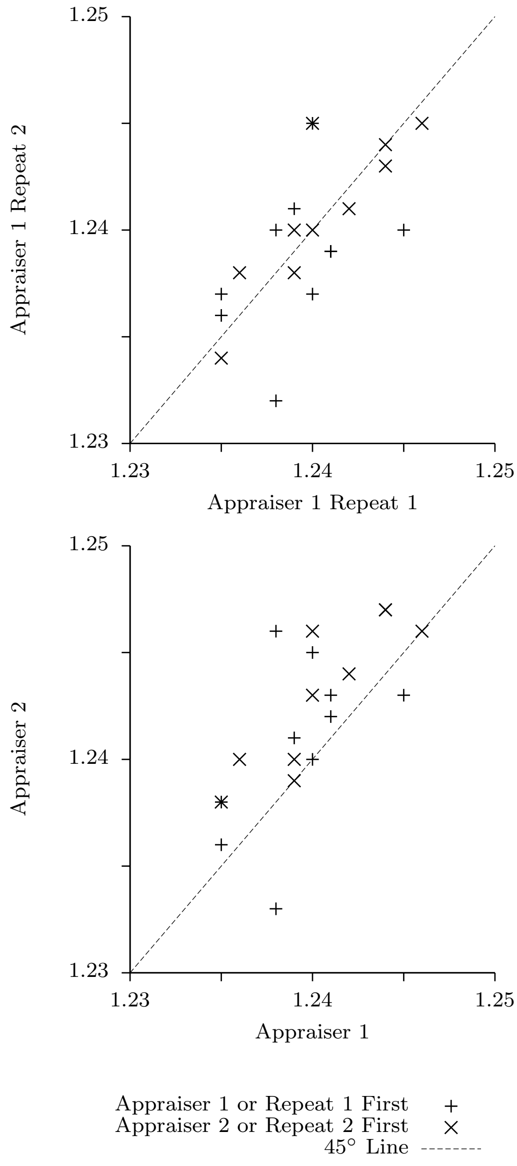


Figure 4.14: Isoplots for sample thickness between (a) appraiser 1 repeat measures and (b) appraisers.

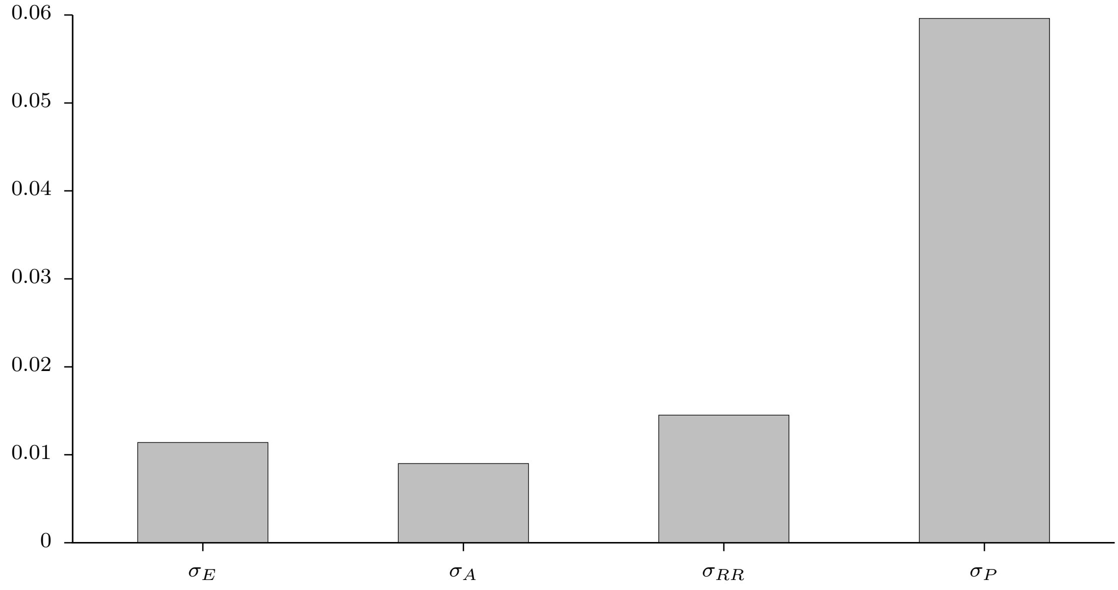


Figure 4.15: Gage R&R from Coveris data.

piece variation which is caused by a non-uniformity across each sample and is not caused by excessive measurement variation.

#### 4.6.3.7 Process Capability

The final PROVADT analysis of the provisional process capability aided the investigation to put the overall variation in the Erema process into perspective. The capability indices for all of the locations measured are $C_p^{(1)} = 5.89$, $C_p^{(2)} = 3.67$ and $C_p^{(3)} = 4.01$, and $C_{pk}^{(1)} = -3.32$, $C_{pk}^{(2)} = -1.82$ and $C_{pk}^{(3)} = -1.17$.

The $C_p$ results of greater than 3.5 indicate that the overall variation in the process is very low with respect to the tolerances. However, the negative $C_{pk}$ results are indicative of a process operating outside of its tolerances. These variation spreads are plotted in Figure 4.16, and from this it can be confirmed that the overall process is outside its tolerance. If original PROVADT had been used, the $C_p$ and $C_{pk}$ values would have only been calculated in two rather than three locations.

#### 4.6.3.8 Overall Analysis Provided to Coveris

As a result of this overall analysis a single control parameter that affected the flow of plastic extruded was identified, enabling the re-centring of the process. Further action was taken to establish set-up protocols to ensure process operators adjusted this parameter so the process remained on target.

Further investigations were undertaken to identify and optimise parameters which effect the product-within-piece variation. These gains ultimately lead to Coveris minimising the quantity of plastic used to produce plastic reels, thus leading to financial savings, whilst also

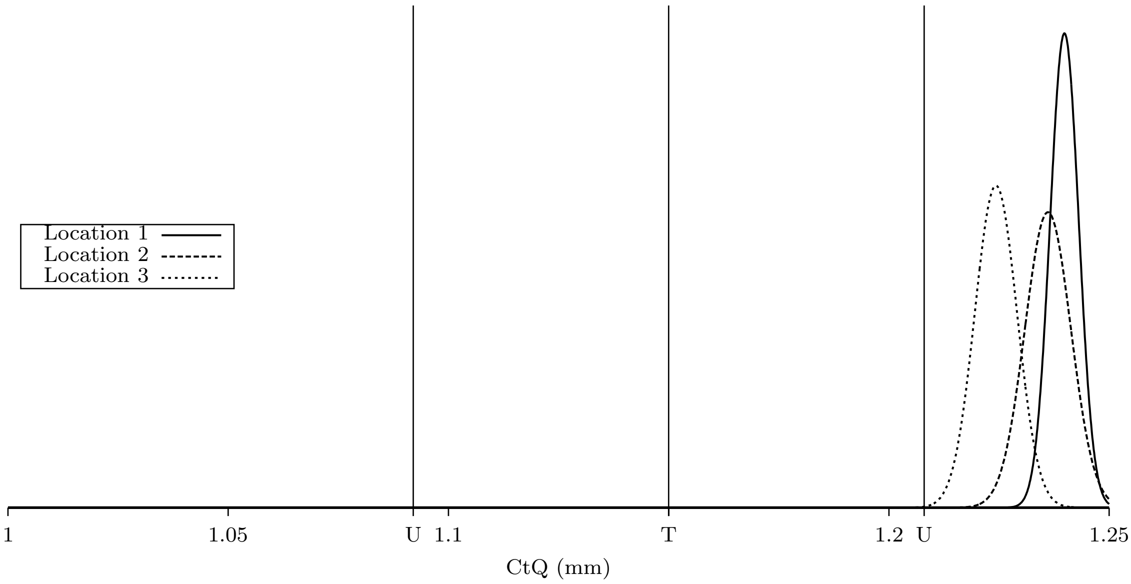


Figure 4.16: Provisional process capability from Coveris data.

providing its customers with a more consistent product.

## 4.7 Summary

This chapter has illustrated the application of an updated PROVADT sampling plan [74]. The enhanced PROVADT approach addresses a compromise in the original method of not always providing a true Gage R&R. The benefits of incorporating a true Gage R&R enables the assessment of whether within-piece variation is caused by measurement variation or product variation. This new sampling procedure now allows a practitioner to apply a Multi-Vari chart, Isoplots, Gage R&R and provisional process capability study to one set of samples and measurements. From these analyses, the signature of the dominant cause of variation is diagnosed and the scale of process variation is contextualised.

The enhanced PROVADT procedure narrows down the focus of a quality improvement project. It moves the project from the measure to analyse phase, in the DMAIC cycle, using passive experimentation, that does not disturb an on-going process, and narrows the focus using objective, data driven analysis.

The application of the approach has also been outlined, using both a worked example from a simulation and two real industrial case studies. The flexibility of the approach was demonstrated in the case studies, with one company producing radiators in a discrete process and the other producing extruded plastic in continuous batches. In both cases it was possible to measure within-piece in multiple locations, by identifying sufficient time periods to assess the process variation and capturing consecutive samples. The follow-up analysis, in both cases, drove the projects forward significantly in the search for the dominant cause of variation in their processes.

# Chapter 5

# Set-Up Process Algorithm (SUPA)



## 5.1 Introduction

This chapter is based around material published by Cox et al. [123, 124, 125]. It was identified in chapter 1 that AMTs, such as machine tools, are increasingly required to operate in a low-volume, high-variety production paradigm. This typically creates a pattern of variation in a process known as *'set-up dominance'*; whereby, the process is highly-capable over a production run and the largest source of variation is between production batches. This variation, ultimately, appears in a product's CtQ features and when these CtQs are required to be produced with high-precision, managing this set-up dominant variation becomes critically important.

As described in chapter 3, in a traditional mass-production environment process variation is monitored using a suite of statistical tools known as SPC. Included in SPC are tools such as: $\bar{X}$-chart, for monitoring process mean; $s$-chart, for monitoring process standard deviation; and $R$-chart, for monitoring process range. These tools are used to identify when a statistically significant change has occurred in a process, so that corrective action can be taken before the process goes out-of-tolerance with respect to a products CtQs. However, in a low-volume production paradigm these tools are not applicable, resulting in low-volume manufacturing processes being controlled by operators using their *intuition* or *rules-of-thumb*.

It has been shown that this approach increases variation rather than reducing it [80].

This problem can be avoided if a defect prevention, rather than statistical, approach to process control is taken. Defect prevention process control tools typically set control limits with respect to CtQ design tolerances, rather than process performance and capability. They have a goal of identifying when a process is not performing to a predefined minimum capability standard. These tools can be split into two groups parametric and non-parametric; whereby, parametric tools make distributional assumptions about a processes and non-parametric tools make no distributional assumptions. The literature on this topic was reviewed more completely in section 3.4.

A case study is described in the following section to provide context of an industry situation where a low-volume process control tool is applicable. A new non-parametric defect prevention approach to process control, known as Set-Up Process Algorithm (SUPA), is defined in section 5.3. In section 5.4 an experimental methodology of using a stochastic discrete-event simulation of a low-volume production environment to test the performance of process control tools is explained. Results from this experimental methodology to compare the performance of SUPA against other SPC and defect prevention tools, outlined in the literature as suited for low-volume control applications, are detailed in sections 5.5 and 5.6.

## 5.2 Case Study I

### 5.2.1 Introduction

This case study was conducted at a manufacturer and global supplier of quality metal printing machinery. In the manufacturing facility there are numerous low-volume processes producing in excess of 5000 unique parts in batches as low as 5 units. The company did not employ any SPC methods, as it was perceived that they were not applicable in low-volume processes. This section highlights the current practices employed to control set-up dominant machine tools, the perils observed that result from these procedures and the practical obstacles to deploying an SPC methodology.

This study focused on current working practices around a Nakamura TW-25 CNC twin turret lathe machine tool. The company used this precision process to machine pre-cast billets into gear blanks. Gear blanks of multiple shapes and sizes are produced in low-volumes, to fulfill the assembly and service requirements of multiple product varieties. These gear blanks go through further processes to produce a final gear cog, which is used in a drive assembly of a final metal printing machine. At this stage, the gear blank has two CtQ features, which are its inner-bore diameter (ID) and its outer-diameter (OD) as shown in figure 5.1.

In this process the control parameters used to adjust the position of a CtQ were the machine tool offsets. It was known by process operators that there was independence between machine tool offsets and the part's CtQs. This means adjusting one offset has a direct effect on a single CtQ.

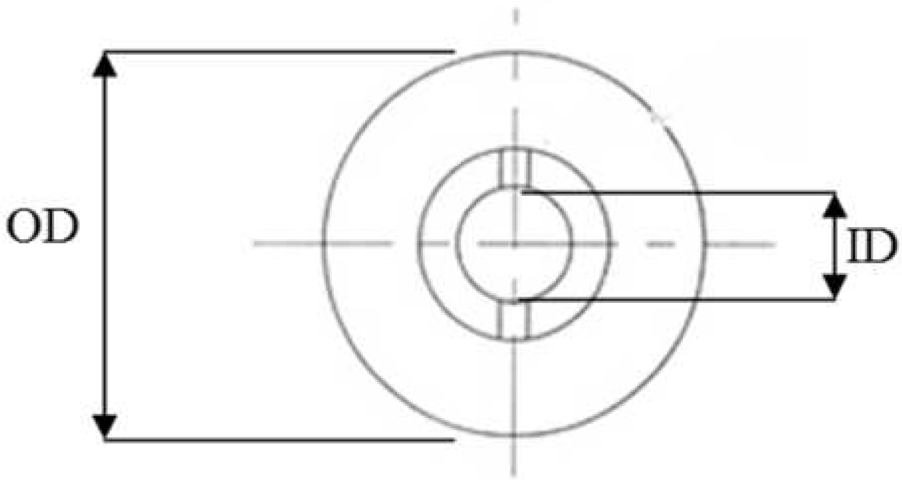


Figure 5.1: An example gear machined on a Nakamura TW-25 with ID and OD denoted.

### 5.2.2 Current Practices

#### 5.2.2.1 Standard Set-Up Procedure

To commence a new production run, a TW-25 machine operator collects a batch card detailing the part with engineering drawings and production volume. The operator proceeds into the machine set-up for the run, this takes 120 minutes on average irrespective of the number of parts to be manufactured. The set-up process involves the operator downloading the correct Computer Numerical Control (CNC) program onto the TW-25 for the part, loading the correct tooling into the TW-25 and collecting the correct size billets.

Once the machine set-up is complete, a billet is loaded into the machine and a *'test cut'* is made. The test cut is pre-loaded in the CNC program and is set 0.5mm outside of the CtQ tolerance, i.e. as little material is removed from the billet as possible. The OD and ID are measured with micrometers and the offset from the design target calculated. This offset is used to adjust the TW-25 to cut within tolerance. The work piece used for the test cut is reworked with the TW-25 calibrated with the new offsets; this is checked again with micrometers to ensure it is now within tolerance. Subsequent parts then have their ID checked with a go/no-go gauge and the OD checked with a micrometer. The frequency of these checks is largely dependent on the operator managing the process. Although operators were expected to produce in-tolerance parts, the responsibility of *'quality control'* was left to an inspection team who checked parts CtQs conformance between processes.

#### 5.2.2.2 Observed Practices

Beyond the standard set-up procedures stated, the management of the TW-25 process was largely dependant on the operator, this led to a range of actual practices. An example, was that operator A overrides the CNC offsets whilst making his test cuts. He reduced the pre-set of 0.5mm to 0.2mm and cites an improvement in machine precision in the subsequent cut. He believed that reworking a piece with a 0.5mm test cut leads to excessive tool deflection in the next cut. This does call into question whether a test cut, whilst working with precision machine tools, was needed at all. Testing this theory was outside the scope of this project, which is to look at the effects of process controls. However, it is an example of how a vague Standard Operating Procedure (SOP) can lead to variation in operator actions, which in turn, leads to variation in product batches made by different operators.

This operator to operator difference did not stop with test cuts; another observed difference was the approach to in-process inspection and in this regard there were two clearly different approaches. The first approach was to measure each part produced and adjust the process when a part's CtQ was out-of-tolerance. Some operators felt this approach was too time consuming and it was not their responsibility to check every part given that there was an inspection team. This led to the second approach of ensuring the first part is in tolerance and then letting the process run; thus, reducing the in-production inspection time. However, neither of these approaches contain statistical rigour. In the first instance, changes are only made to a process once it is out-of-tolerance; therefore, making reactive changes, rather than preventative ones. In the second instance, the process is allowed to run once one part is in-tolerance; but, this does not give confidence that the process is currently capable of producing a batch of in-tolerance parts. Operators adopt this approach citing that it saves time, but in this low-volume production the most significant time overhead is the machine set-up. If a part is revealed to be out-of-tolerance by post-production inspection, this leads to a second set-up of the TW-25 machine to rework or reproduce scrapped parts. This has a significant impact on the processes capacity.

An observation that appeared consistent across all operators, was the aim to make parts closer to the top tolerance. From an operator perspective, their job is to produce parts with CtQs that are within design tolerances. If an out-of-tolerance part is produced at the bottom tolerance, i.e. too much material has been removed, this part is scrapped. If an out-of-tolerance part is produced at the top tolerance, i.e. too little material has been removed, this part is reworked to bring it in-tolerance. Therefore, operators perceived it to be beneficial to set-up the TW-25 to make parts just in-tolerance to reduce the chance of scrap.

However, the design team took a statistical approach to the design of tolerances; whereby, there is an expectation that on average parts are manufactured with CtQs that are on-target. This conflicts with the current manufacturing's approach, were parts are produced inside the CtQ's top tolerance, leading to persistent tolerance stacking issues in assembly and servicing. For example, if a gear cog ID is machined to the top of its tolerance this results in an acceptable CtQ with a minimum bore size. If a shaft, which the gear is to be mounted onto, is also manufactured to the top of the tolerance this makes it have the maximum outer diameter. There is then potential for at least two interfacing part to be both intolerance, but not fit together. This tolerance stacking issue was appearing both in final assembly of

Table 5.1: List of parts used for trial, machine processed on and date of processing.

| Part | Machine | Production Date |
|---|---|---|
| G99-664A | TW25 | 12/10/2011 |
| G99-285 | TW25-B5514 | 13/10/2011 |
| G99-161 | TW25-B5514 | 17/10/2011 |
| G99-663A | TW25-B5514 | 17/10/2011 |
| W99-2780 | TW25-B5514 | 18/10/2011 |
| W65-8487 | TW25-B5518 | 18/10/2011 |

new metal printing machines and in the servicing of customer machines. To combat this, the design team were constantly tightening CtQ tolerances; which in turn, was making parts increasingly difficult to manufacture. In order, to determine the potential of using a defect prevention approach to SPC in a low-volume process, a trial was run on the TW-25 using Pre-Control which is discussed in the next section.

### 5.2.3 Pre-Control Trial

#### 5.2.3.1 Introduction

The defect prevention approach to SPC of Pre-Control, described in chapter 3, was trialed by operators of the TW-25. The object of this trial was to identify if a defect prevention approach to SPC was applicable in a low-volume production environment. Also, limitations identified at this stage would inform further research.

To establish this trial, six different parts were selected that contained ODs and IDs. The batches that the six different parts were to be produced in, were of sizes ranging between 8-12 parts. These were due to be produced during the week commencing 10/10/2011, but due to changes in production scheduling the trial lasted two weeks. The parts used and production dates are listed in table 5.1.

As the focus of the company's machining operations was in low-volume production, the trial used phase 1 of Pre-Control which requires five greens to validate the process. Further to this, a percent tolerance Pre-Control approach, described by Vermani [126], was employed so that a custom chart did not need to be produced for each part and each CtQ on that part. Examples of the record sheet and percent tolerance Pre-Control charts used can be found in appendix D. In order to convert a measured value of CtQ into a percentage, operators were required to use the following calculation:

$$X_{PC} = \frac{100(X - T)}{0.5(U - L)}, \tag{5.1}$$

where $X$ is the actual dimensional measurement and $X_{PC}$ is the associated percent tolerance Pre-Control value. The $X_{PC}$ value can then be plotted on the traffic light percent tolerance Pre-Control chart. These charts are in appendix D.

In order to minimise the effects of different test cut sizes between operators, a standard test cut offset of 0.2 mm was applied by all operators. The operators had to use micrometers on both OD and ID features, to collect variable data. Also, operators were required to adjust an off-target process towards its design target, rather than just inside its top tolerance. An exploration of the results from this trial is given in the next section.

#### 5.2.3.2 Results

The records and percent tolerance Pre-Control charts collected during this trial are presented in appendix D. The first observation, is that in all cases both the OD and ID features were validated. This means that five consecutive green parts for each feature were collected before the end of the production run. Using the rationale described in chapter 3, this gives the process operator 98% confidence that the process is both on target with a minimum capability of $C_P \geqslant 1.33$. Although this is a limited study, it suggests that the process is capable of manufacturing design CtQ features with a high capability.

It was also noticeable, that in some cases the first adjustment made after the test cut was not always sufficient to bring the process on-target, but it was sufficient to bring the process in tolerance. Traditionally these processes would have been allowed to continue; but using Pre-Control when there was a pattern of two consecutive yellow units on the same side of the process target, the operator received a signal to make a further adjustment. After these adjustments the process was consistently validated, with five green units. This demonstrates that having a process control methodology across parts and operators aided consistent control actions. This led to processes which are on-target and not just within tolerance.

These results do raise one question, are these processes more capable than a minimum of $C_P \geqslant 1.33$? If they are and a higher capability is desirable from the point of view of design, can the Pre-Control method be modified to allow for this?

From the point of view of implementation, there was an issue with the data collection process. Using a paper-based records and chart system, which required operators to convert actual measurements into a percentage Pre-Control value, delayed processing times. Further to this, the operators in the trial felt that if a paper-based system was rolled-out, some operators would not fill out the forms correctly to avoid the perceived hassle of making calculations. A solution to this could be to use an electronic data collection system, which is seen in high-volume SPC implementations. However, further research into the cost and implementation in low-volume applications would be required.

### 5.2.4 Conclusion

This case study identifies shop-floor practices, such as making parts inside tolerance rather than on-target and inconsistencies between operator adjustments of processes. These problematic practices are consistent with the issues identified in the literature, in chapter 3.

A defect prevention approach to SPC was trailed, as it was identified in section 3.4 of the literature review as a potential solution. Using Pre-Control did improve the consistency of operator process adjustments and also forced the operators to adjust the process to its design target. However, it did raise a research question: can phase 1 Pre-Control be adjusted when a capability of $C_P \geqslant 1.33$ is desirable? This question is explored in the following sections of this chapter.

## 5.3 SUPA Definition

### 5.3.1 Introduction

In this section, a process control method known as SUPA is outlined. The research that led to this method was conducted in response to rule-of-thumb set-up practices conducted by operators of low-volume manufacturing processes. These rule-of-thumb practices have been identified in both the literature, see chapter 3, and in the case study detailed in the previous section.

In the previous section phase 1 PRE-Control, was identified as a process control tool suitable for low-volume manufacturing processes. It was identified as a suitable tool due to it:

- providing operators with an easy to define and clear control chart based on product CtQ tolerance;
- providing operators with a clear set of rules to identify when a process is conforming to a minimum standard of $C_p \geqslant 1.333$;
- identifying with 98% confidence (i.e. 2% $\alpha_q$-risk) that the process is conforming to this standard;
- distinguishing between a non-conforming process that is off-target and a non-conforming process that has excessive variation.

However, phase 1 PRE-Control has a clear issue with respect to high-precision processes. For example, is a minimum standard of $C_p \geqslant 1.333$ good enough? If a precision process has a high $C_p$ does PRE-Control ensure the process is centred? In a world where world-class manufacturers are aiming for $C_p \geqslant 2.0$, a process control tool that maintains a minimum standard of $C_p \geqslant 1.333$ is certainly not good enough.

The issue is not as clear-cut as tweaking PRE-Control to maintain a new standard of $C_p \geqslant 2.0$. There are different features of the method that are adjustable, as identified in section 3.4. These include: the number of consecutive yellow units needed to identify a nonconforming process ($t$); the number of consecutive green units needed to validate a process ($k$); the probability of qualifying a valid process ($P(q)$) where $\alpha_q = 1 - P(q)$; and the width of the central green band ($G_U - G_L$). Adjusting each of these features has different effects on the final performance of a process control tool. A final SUPA method definition is described next, by selecting values of $t$, $k$ and $P(q)$ and using probability theory outlined in section 3.4.

### 5.3.2 Method

To define a final SUPA method, the first step requires the selection of the $t$ and $k$ parameters. An aim of the method is to provide operators with an easy to implement protocol to use on the shop-floor. Therefore, the values of $t$ and $k$ remain constant to increase the practicality of the method. This means that an operator moving between processes has a consistent set of rules to follow. The established rules of phase 1 PRE-Control, have been shown to be practical in the literature (section 3.4) and in the previous case study. Hence, SUPA uses these rules (table 5.2), which results in $t = 2$ and $k = 5$.

Table 5.2: Outline of SUPA decision rules for $t = 2$ and $k = 5$.

| Sampled Units | Observation | Action |
|---|---|---|
| 1 | Red Unit | Stop and Adjust |
| 1 2 | Two Consecutive Yellow Units Same Side of Target | Stop and Adjust |
| 1 2 | Two Consecutive Yellow Units Opposite Sides of Target | Stop and Investigate |
| 1 2 3 4 5 | Five Consecutive Green Units | Continue Process |

Based on these values of $t = 2$ and $k = 5$, the generalised phase 1 Pre-Control formula (equation (3.20)) is reduced to:

$$P(q) = P(g)^5 \cdot \frac{1 + P(y)}{1 - P(y)\sum_{i=1}^{4} P(g)^i}. \tag{5.2}$$

The next value to define is $P(q)$, the confidence that a process is on-target and capable. As outlined in the literature review, $P(q) = 1 - \alpha_q$; whereby $\alpha_q$ is the risk of a false fail. The classic criticism of phase 1 PRE-Control is its 2% $\alpha_q$-risk is too high. For example, classic SPC tests such as $\bar{X}$-chart, have an $\alpha_q$-risk of 0.27%. However, if the $\alpha_q$-risk is reduced below 2% in SUPA, this has the effect of widening the central green band, see [111]. This has the effect of allowing an increased number of off-target processes being incorrectly validated. Also, when compared to current practices in precision machining processes of using rule-of-thumb methods to validate, an $\alpha_q$-risk of 2% is low. Therefore, SUPA maintains the use of an $\alpha_q$-risk= 2%, and $P(q) = 0.98$.

SUPA maintains the intuitive linkage of the red zones to design tolerances, which means there are only two remaining parameters can be modified: minimum $C_p$ and the width of the green zone ($G_U - G_L$). Given the aim of SUPA is to provide a traffic-light chart based method of process control that is adjustable to maintain different minimum standards of $C_p$, this is directly controlled by adjusting the width of the green zone. To find green zone widths that maintain a $P(q) = 0.98$, different values of $P(g)$ and $P(y)$ are calculated by:

$$P(g) = \Phi\left(\frac{G_U - \mu}{\sigma}\right) - \Phi\left(\frac{\mu - G_L}{\sigma}\right), \tag{5.3}$$

and,

$$P(y) = \Phi\left(\frac{U-\mu}{\sigma}\right) - \Phi\left(\frac{G_U-\mu}{\sigma}\right) + \Phi\left(\frac{\mu-G_L}{\sigma}\right) - \Phi\left(\frac{\mu-L}{\sigma}\right). \tag{5.4}$$

A range of green zone widths, described as a percentage of the design tolerance, can then be correlated for given minimum standards of $C_p$. These results are plotted in figure 5.2.

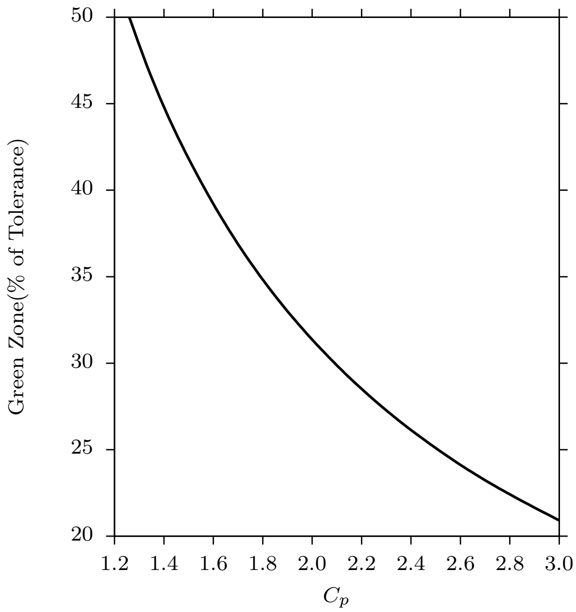


Figure 5.2: SUPA percentage green zone against minimum $C_p$ at $P(q) = 98\%$ confidence.

These green zone widths are then set for each CtQ of a product by an operator on the shop-floor or at design level. The procedure to set up a SUPA-chart is described by an algorithm shown in figure 5.3.

The next section will give a worked example of how to set up and use the SUPA method.

### 5.3.3 Worked Example

#### 5.3.3.1 Chart Preparation

In this section, the application of SUPA is illustrated. The example used in this section, considers a CtQ that has a $T = 100$, $U = 200$ and $L = 0$. The process which machines the CtQ is required at a design level to have a minimum $C_p \geqslant 2.0$. Using figure 5.2, it is estimated that the green zone should be 31% of the design tolerance, to ensure a minimum $C_p \geqslant 2.0$. The resulting SUPA-chart, figure 5.4, has Red zone limits fixed at the design tolerances of $U = 200$ and $L = 0$, and also has Green zone limits fixed at $G_U = 131$ and $G_L = 69$.

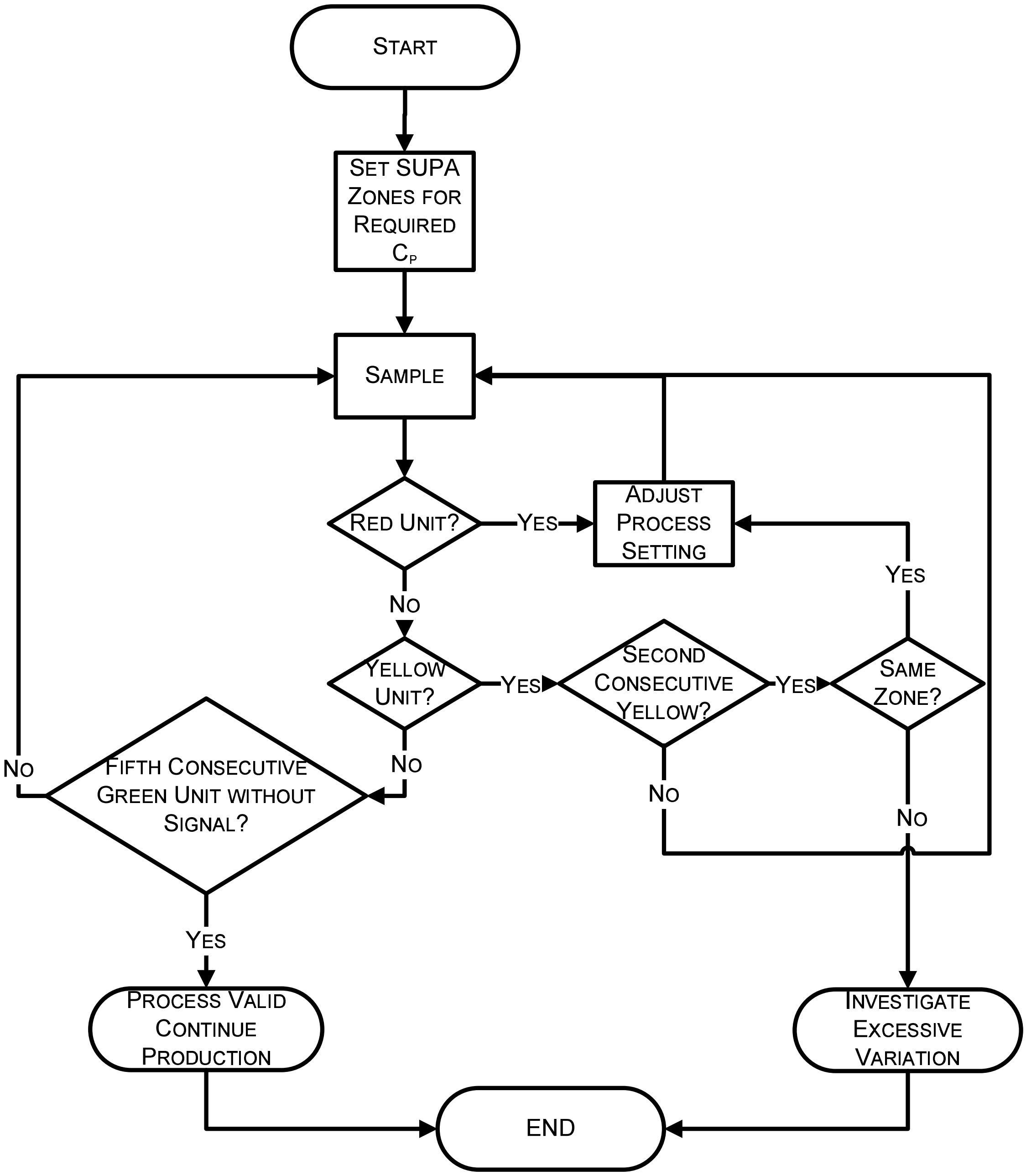


Figure 5.3: Flow chart of SUPA algorithm.

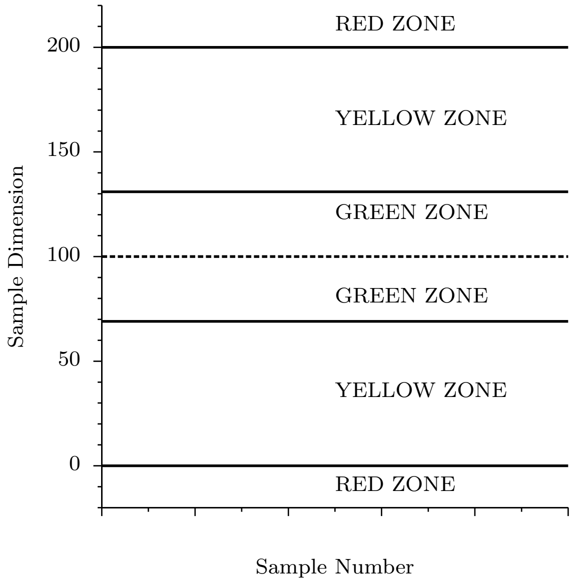


Figure 5.4: Configuration of a SUPA-chart.

The following sections demonstrate how a SUPA-chart detects if a process is: off-target or incapable.

#### 5.3.3.2 Process Off-Target

Once the SUPA-chart has been established, production of the example part can begin. The data collected in Table 5.3 shows the measurements of the monitored CtQ. The results in Table 5.3 are plotted on a SUPA-chart in figure 5.5.

Table 5.3: SUPA run chart of example results.

| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 135.0 | 135.0 | | | | 105.0 | 105.0 | 105.0 | 105.0 | 105.0 |
| 2 | | 147.5 | | | | | 97.5 | 97.5 | 97.5 | 97.5 |
| 3 | | | | | | | | 112.5 | 112.5 | 112.5 |
| 4 | | | | | | | | | 85.0 | 85.0 |
| 5 | | | | | | | | | | 95.0 |
| action | | -41.25 | | | | | | | | valid |

In the first stage of production, the process was off-target. The SUPA-chart in figure 5.5 identified this after the first two sampled products, which were both in the yellow zone on the same side of the tolerance. This indicates that the process is off-target and needs adjustment (Table 5.2). It is indicated in the record in Table 5.3 that an adjustment of -41.25 was made to the process.

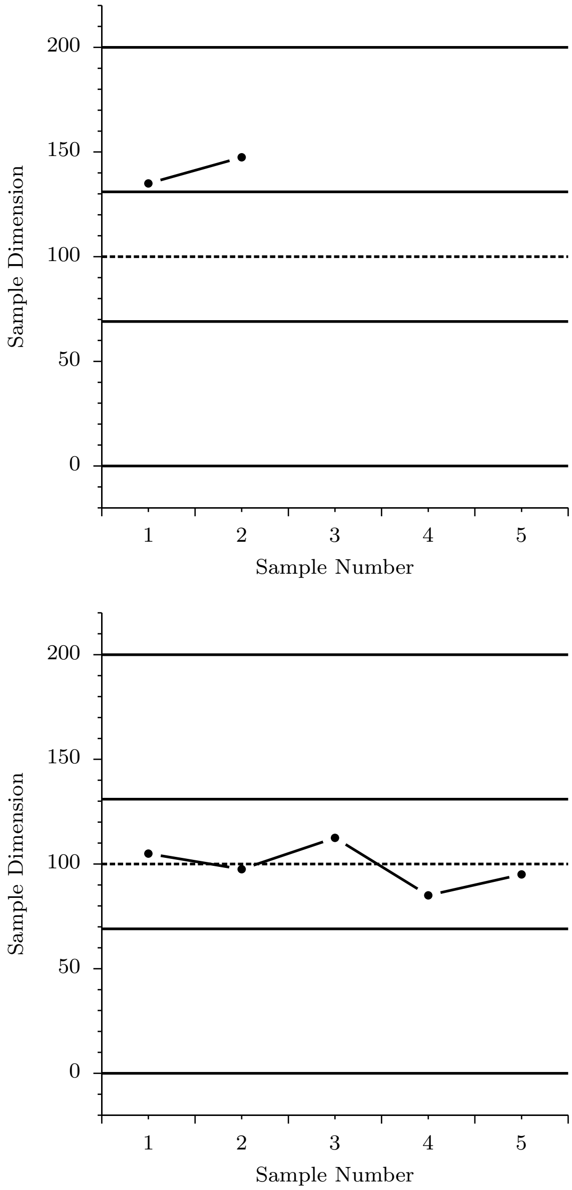


Figure 5.5: SUPA-charts of example results: a) at the start of a new process set-up, and b) after the introduction of a process adjustment.

In the second stage, after the corrective adjustment was made, the process produced five consecutive parts. For the SUPA-chart, it highlights the process's validity, because five consecutively produced part's CtQ fell inside the Green zone. This indicates that the process is both on-target and capable, i.e. it is not producing parts with excessive variation.

#### 5.3.3.3 Process Incapable

In this second example of the same process, two samples are taken shown in Table 5.4.

Table 5.4: Example run chart

| | 1 | 2 |
|---|---|---|
| 1 | 135.0 | 135.0 |
| 2 | | 63.0 |
| action | | **Investigate** |

Using the SUPA-chart in figure 5.6, it can be seen that the two samples fall in yellow zones either side of the green zone. This indicates excessive variation, according to the rules outlined in Table 5.2. Therefore, the process was deemed to be incapable, i.e. there was too much product variation. When there is an indication that there is excessive variation present, further investigation is required to determine the cause.

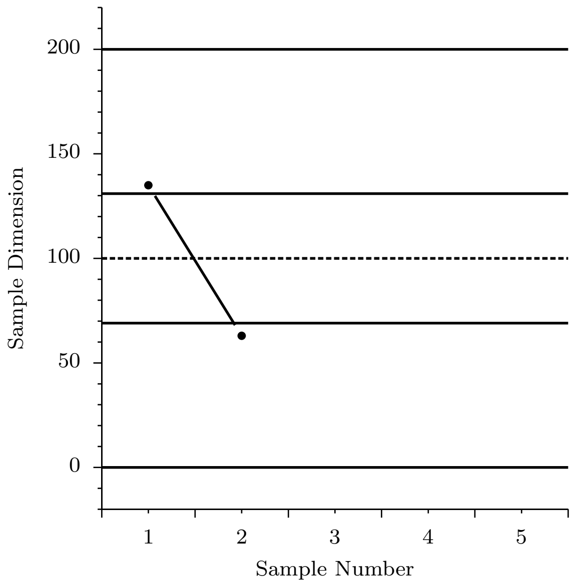


Figure 5.6: SUPA-chart of example results for the start of a new process set-up, showing excessive process variation.

### 5.3.4 Summary

This section has described a new process control methodology to monitor sets-up dominant processes. The performance of the method known as SUPA was tested using a discrete-event simulation model. This model is described in the following section. The model is then used in sections 5.5 and 5.6 to test the performance of SUPA against other process control methodologies in a set-up dominant environment.

## 5.4 Performance Testing Methodology

### 5.4.1 Introduction

In this section a general discrete-event simulation model is described that can be used to test the performance of different process control tools in a set-up dominant environment. Some performance characteristics of process control methods, such as probability of qualifying, can be analytical derived. This does not show how a process responds to the adjustments implemented by the respective methods or the number of units needed to validate the process. Since there are no detailed analytical models for that, a discrete-event simulation model was utilised. The general model is outlined and then validated against performance characteristics that can be determined analytically. This validation gives confidence in the performance results obtained through simulation that cannot be derived analytically. This general discrete-event simulation model is then used to examine the performance of four specific process control tools in a set-up dominant environment in sections 5.5 and 5.6. The process control tools analysed where selected based on the review in section 3.4 and include: SUPA, Pre-Control, Acceptance Control Chart (ACC) and SB$\bar{X}R$.

### 5.4.2 General Discrete-Event Simulation Model

In order to further test the effectiveness of the different methods of validating the set-up of a process, a discrete-event simulation model was built using WITNESS (Lanner Group Ltd., UK). The model simulated a generic process applying a CtQ to a unit. For example, this could represent a lathe machining the outer diameter of a gear. The process has a $U = 200$ and $L = 0$ and a process target, $T = 100$. The current process mean, $\mu$, can be offset at the start of the simulation. The model adjusts $\mu$ based on the decision rules of the control method analysed. Capability is set prior to the simulation and remains constant throughout.

The simulation model applies adjustments by finding the mean of the units signalling an adjustment ($\bar{A}$). Then it subtracts the difference between $\bar{A}$ and the process target, $T$, from the current mean ($\mu(k)$) to find the new process mean ($\mu(k+1)$), i.e.

$$\mu(k+1) = \mu(k) - (\bar{A} - T) \tag{5.5}$$

The general model can be seen in Figure 5.7. At the start, the experimenter sets the initial parameters of capability and process mean (boxes 1 and 2). The model is allowed to run. Units enter the model (box 3) with a generic process applying a CtQ to each unit (box 4), based on parameters of capability and process mean. The model then samples consecutive units (box 5). Based on the decision rules of the Control method utilised, a decision is made on whether or not the process is valid (box 6). In the case of SUPA, if there were five consecutive units in the green zone the model will be validated. If a model is validated or deemed incapable, sampling immediately stops (box 7). If the model is not validated, the model decides whether an adjustment is needed to the current process mean (box 8). If an adjustment is not needed sampling continues (box 5). If an adjustment is needed, the mean is recalculated (box 9) by equation (5.5). The adjustment is then applied to the process mean (box 2).

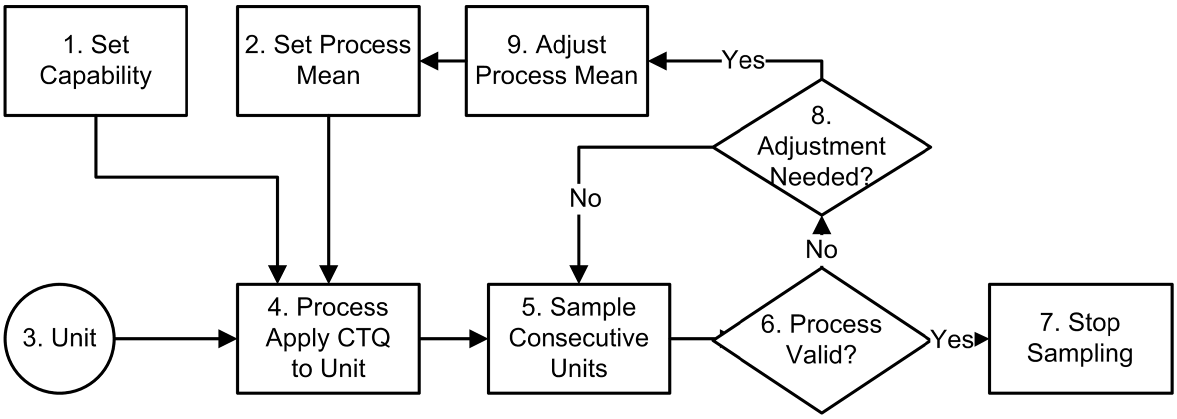


Figure 5.7: Process flow of general discrete-event simulation model.

### 5.4.3 Model Validation

To test the validity of the simulation model, the function of adjusting the process was switched off. The simulation was stopped if it was validated, incapable or in-need of adjustment. From this the simulations estimate of confidence was compared with the statistically derived results. A model built around the general framework in figure 5.7 was developed to test the performance of SUPA in this situation. Figure 5.8 demonstrates the operating curves for the analytical and simulation results, by plotting the probability of qualifying ($P(q)$) the SUPA method against a range of $C_p$ values. It is assumed in this example that the SUPA chart has been set up to maintain a process with a $C_p = 2.0$.

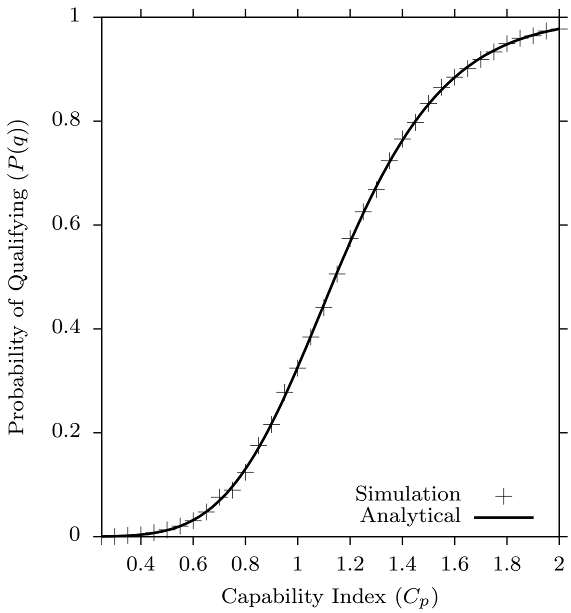


Figure 5.8: Operating curves of SUPA's analytical and simulation performance.

The analytical results for SUPA, shown in figure 5.8, are derived from equations (5.2) - (5.4). It can be seen in figure 5.8 that the difference between analytical and simulation results was small for the SUPA control method. These SUPA results are typical for the other methodologies simulated. This close alignment allows credible results to be derived from the simulations conducted in the following sections.

## 5.5 Parametric Analysis

### 5.5.1 Introduction

In this section the performance of four process control methods are analysed. The first two methods, Pre-Control and SUPA, are nonparametric process control approaches, they do not make any distributional assumptions. The other methods, SB$\bar{X}R$ and ACC, are parametric and assume the process has a Gaussian distribution. Simulating a process with a Gaussian distribution offers a fair starting point for comparing the respective methods. Therefore, this section performs a parametric analysis of these process control methods. This is achieved by simulating a manufacturing process which produces products with a CtQ denoted by $X$, that follows a Gaussian distributed as follows:

$$X \sim N\left(\mu(k), \left(\frac{U-L}{6C_p}\right)^2\right). \tag{5.6}$$

There are two events which can cause a signal from a process control method: a) when a process is on-target but not capable and b) when a process is capable but off-target. Here, the results are presented from when the simulated process control methods are tested under these two conditions.

### 5.5.2 Process On-Target, but not Capable

The respective methods were tested against a process which was initially set on-target. However, each time the simulation was run it had a smaller $C_p$ and therefore larger process variation. The control methods were all set to monitor a process with a $C_p \geqslant 2.0$. During this analysis SB$\bar{X}R$'s and ACC's control limits, where set based on a historical process capability $C_p = 2.0$ and historical mean equal to the design target. SUPAs control limits, where set based on a required $C_p = 2.0$. The first run of the simulation had a $C_p = 2.0$. Each run decreased in the value of $C_p$, to a final value of 0.667. As these discrete-event simulation models are stochastic, each simulation run had 1,000 replications to minimise deterministic effects of the pseudo-random number generation.

Two questions are posed: a) what is the $P(q)$, despite any adjustments made? and b) how many units does it take to make a decision? These are answered in figure 5.9 and 5.10 respectively.

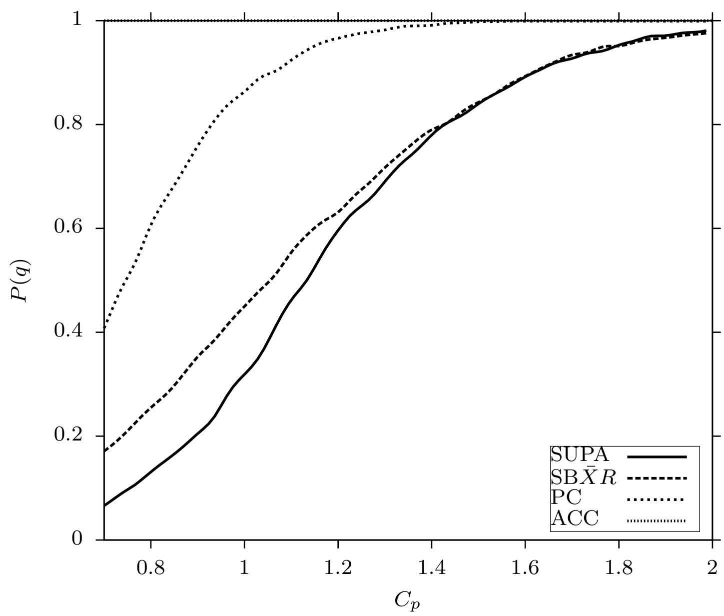


Figure 5.9: Operating curves showing the effect of decreasing $C_p$ against probability of qualifying ($P(q)$).

Figure 5.9 demonstrates some important points:

- ACC has no mechanism to indicate that a process is incapable and eventually validates all set-ups despite the capability. This is indicated by the flat line for ACC at $P(q) = 1.0$ for all values of $C_p$.
- For CtQs which need to be made to a higher $C_p$ than 1.333, SUPA offers significantly improved performance over Pre-Control. This is shown by SUPA having lower values of $P(q)$ as $C_p$ decreases, i.e. SUPA rejects incapable processes more consistently.
- SUPA has better performance than SB$\bar{X}R$ at detecting incapable processes. At a

$C_p = 0.667$ SB$\bar{X}R$ still qualifies over 20% of set-ups whereas SUPA only qualifies 7%.

- These results indicate that SUPA is the most sensitive control method for detecting incapable processes. SUPA has the operating curve with the steepest gradient and as such rejects more invalid processes.

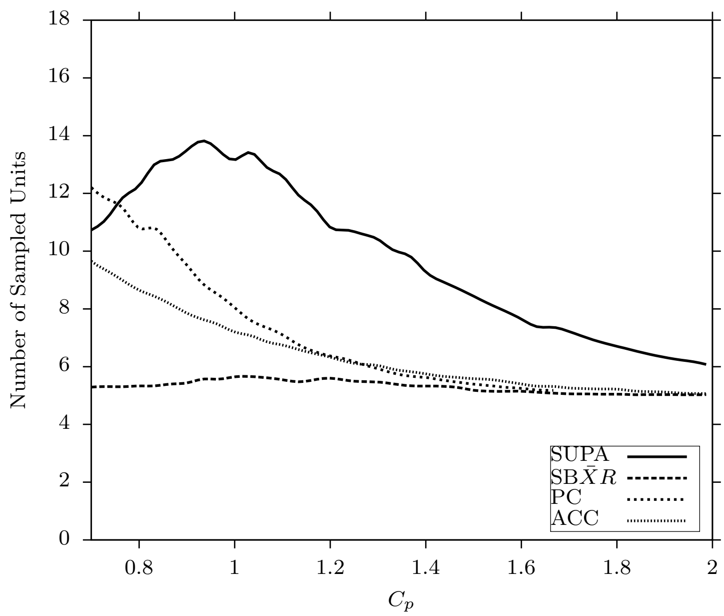


Figure 5.10: Operating curves showing the effect of decreasing $C_p$ against number of samples.

Figure 5.10 highlights that as Cp decreases:

- SB$\bar{X}R$ consistently requires between 5-6 units to reach a final decision.
- Pre-Control and ACC use more units, tending towards 13 and 10 units respectively as $C_p$ approaches 0.667.
- SUPA uses increasingly more units to make a decision, peaking at 14 units.

On the face of these results, SB$\bar{X}R$ appears to be the most efficient method. This result was due to SUPA's decisions being made after a specific sequence of events, whereas, SB$\bar{X}R$'s decisions were made after specific numbers of samples. Using the example of a process with a Gaussian distribution in figures 5.9 and 5.10, and $C_p = 1.0$, it was shown that on average a final valid or invalid decision from SUPA came after 14 units and from SB$\bar{X}R$ after five units. Under these conditions for SUPA, using equations (5.2) - (5.4) shows $P(g) = 0.64$ and $P(y_u) = P(y_l) = 0.17$. Given these probabilities it is possible to imagine how a sequence of samples from a process would *'wander'* between zones without triggering a decision, e.g. from Green zone to upper Yellow to Green to lower Yellow to... before two consecutive Yellows in opposite Yellow zones or five consecutive Greens. Although it is desirable to make a final decision as quick as possible, in the case of an incapable process, additional sampling results in SUPA makes it more powerful at detecting an incapable process. Also, the increase in sampling that occurs as a process becomes less capable has practical benefits, as more defective units are likely to be produced, increased sampling gives protection against out-of-specification parts going undetected.

### 5.5.3 Process Capable, but not On-Target

To assess processes that were capable but not on-target, this analysis tested the respective methods against a process which was initially set with a fixed $C_p$ of 1.333, 2.000 and 2.667. Each time the simulation was run using these settings the process mean was set further away from the on-target state. The process control methods were all set to monitor a process with a $C_p = 2.0$. The first run of the simulation at each setting of $C_p$ started on-target. Following runs began with the process mean, $\mu(k = 0)$, further from the on-target state by a factor, $\varrho$, of the $\sigma$. Again, each simulation run had 1,000 replications.

The most important question in this case was: what was the $C_{pk}$ after a process was validated? This shows how effective the control methods are at centring an off-target process, presented in figure 5.11.

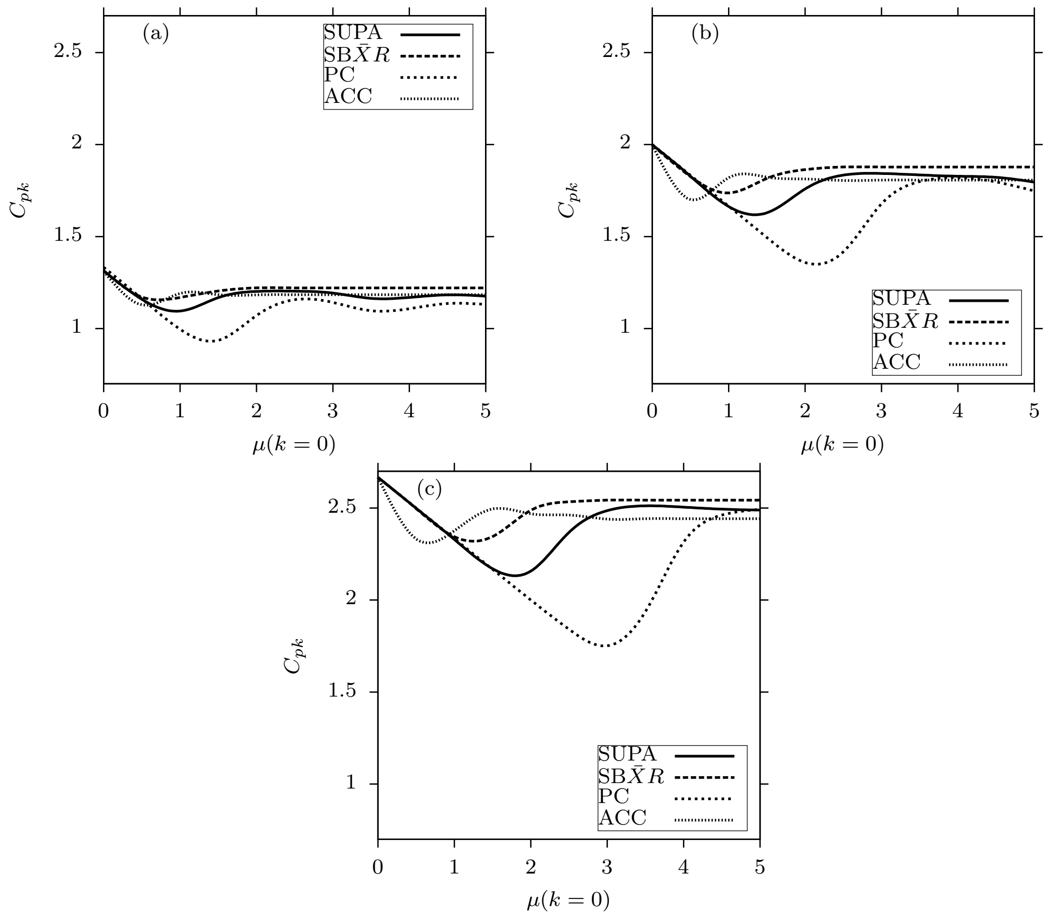


Figure 5.11: Operating curves of final $C_{pk}$ against process mean start positions as a factor of $\sigma$, when the simulation has three different maximum values of $C_p$ = (a) 1.333; (b) 2.000; (c) 2.667, for SUPA, Pre-Control, SB$\bar{X}R$ and ACC.

Figure 5.11 highlights:

- SUPA has improved performance over Pre-Control, maintaining a higher value of $C_{pk}$.
- SB$\bar{X}R$ and ACC have steadier performance profiles over SUPA and Pre-Control. i.e. the operating curves for SB$\bar{X}R$ and ACC more closely follow flat lines.
- SUPA has a dip in performance when a process is one to two standard deviations off-target.

These results highlight that for re-centering an off-target process SUPA offers significantly improved performance over Pre-Control. For example, in figure 5.11(b) where the process has a maximum performance of $C_{pk} = 2.0$, overall SUPA keeps the process $C_{pk} \geqslant 1.63$, whereas, Pre-Control keeps the process $C_{pk} \geqslant 1.36$. Further to this, SB$\bar{X}R$ provides marginally better performance over SUPA. Using the same example, SB$\bar{X}R$ keeps the process $C_{pk} \geqslant 1.7$.

The next question is how many adjustments does each method take to re-center the process? These results are presented in figure 5.12.

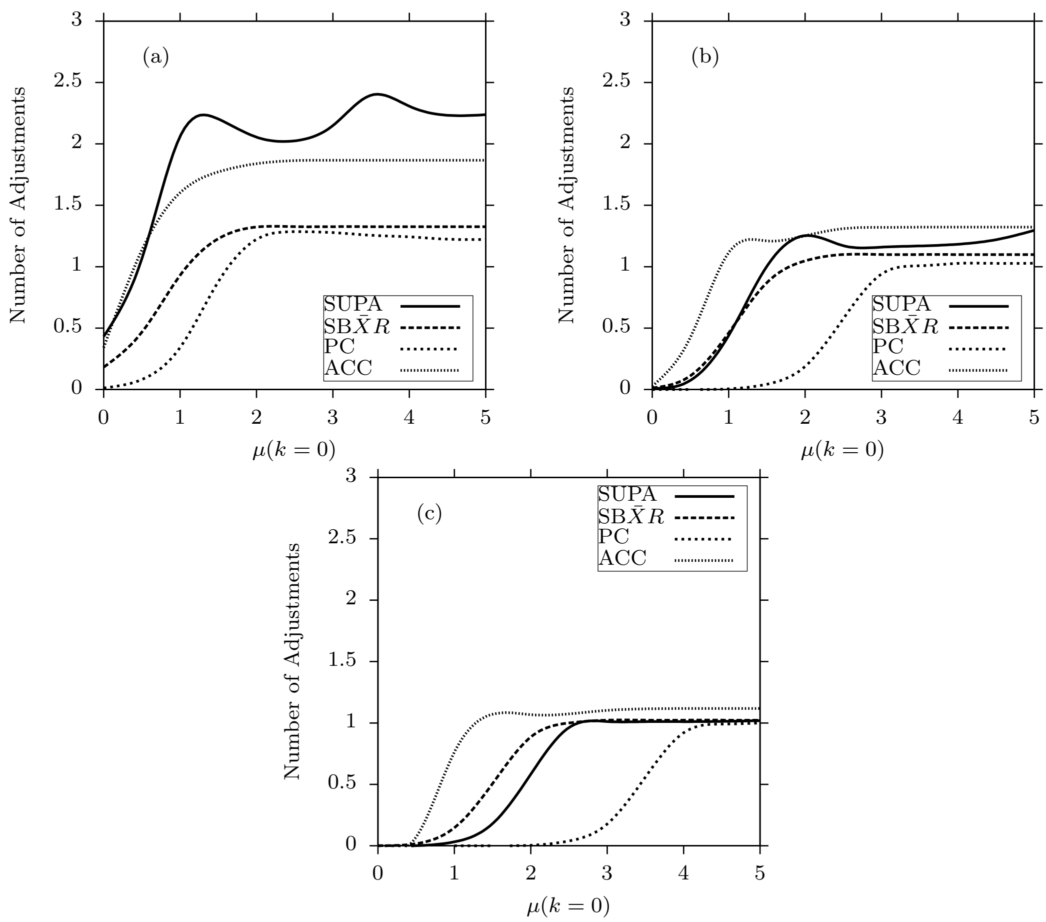


Figure 5.12: Operating curves of number of adjustments against start positions of process mean as a factor of $\sigma$, when the simulation has three different maximum values of $C_{pk} =$ (a) 1.333; (b) 2.000; (c) 2.667, for SUPA, Pre-Control, SB$\bar{X}R$ and ACC.

Figure 5.12 highlights:

- Pre-Control used the least number of adjustments for all values of maximum $C_{pk}$.
- SUPA and SB$\bar{X}R$ use a similar number of adjustments when the process maximum $C_{pk} \geqslant 2.0$.
- SUPA makes more adjustments than any other method when the maximum $C_{pk} = 1.333$. This ties in with the high number of units sampled shown in figure 5.10, when a process has a low $C_{pk}$ but is on-target. Therefore, SUPA needs to make a couple of adjustments to get process on-target before it can deem it as incapable.

Another question is how many samples does each method take to finally validate the process? These results are presented in figure 5.13.

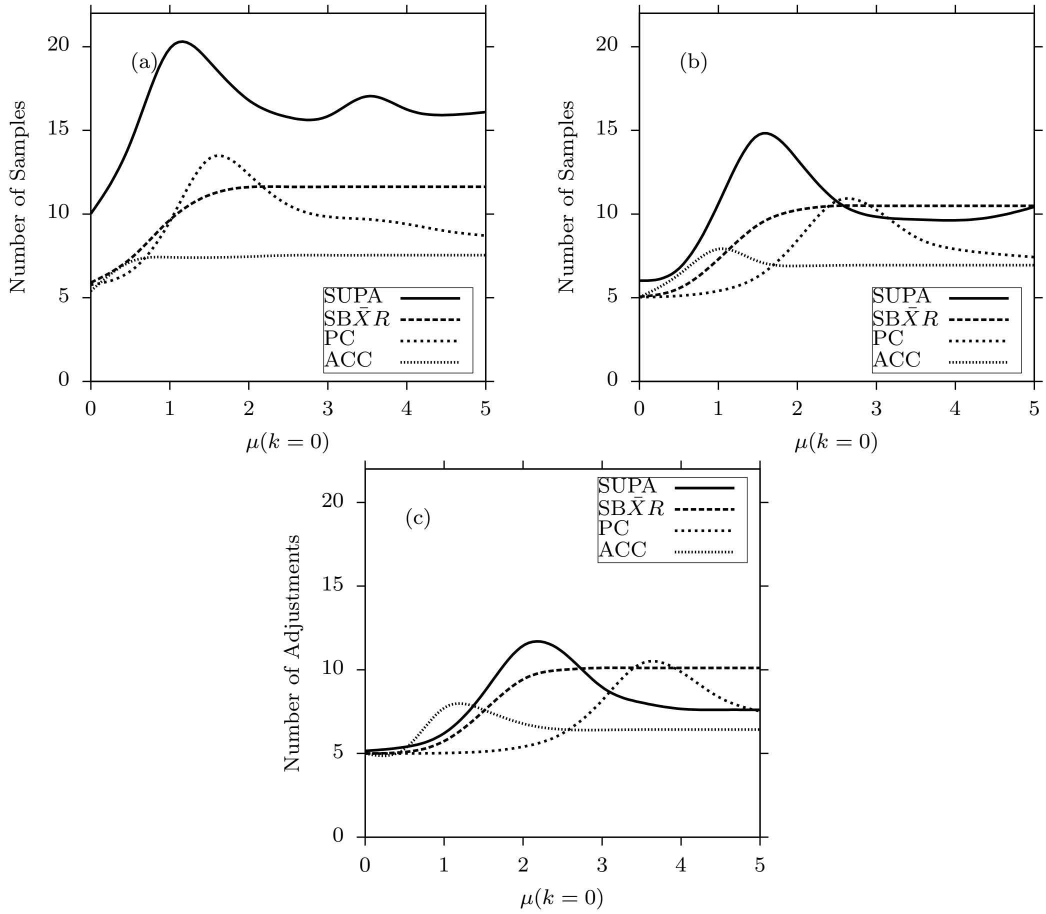


Figure 5.13: Operating curves of number of samples against process mean start positions as a factor of $\sigma$, when the simulation has three different maximum values of $C_p =$ (a) 1.333; (b) 2.000; (c) 2.667, for SUPA, Pre-Control, SB$\bar{X}R$ and ACC.

Figure 5.13 highlights:

- Once the process is off-target, SB$\bar{X}R$ uses approximately 10 samples to get the process back on-target and validate it.
- SUPA uses a similar number or less samples than SB$\bar{X}R$, except in a window where

the process is one to two standard deviations away from its target. For example, if the process has a maximum $C_{pk} = 2.0$, SUPA can use up to 15 samples to validate.

- ACC and Pre-Control generally use less samples than SUPA and SB$\bar{X}R$, this however, is at the expense of performance in other areas.

### 5.5.4 Summary

This section has presented a parametric analysis of four different statistical approaches to process control for set-up dominant processes. The new method known as SUPA was compared with existing industrial practices and statistical techniques in the literature. To test the method's robustness, a generic discrete-event simulation model was used to perform the analysis.

SUPA offers a method of process control for set-up dominant processes, which is easier to apply than classically derived SPC approaches. This is achieved by using simple rules and a traffic light system based on design specification, rather than control limits based on estimated process performance. Simulation analysis also shows that SUPA is more sensitive than other approaches at detecting an incapable process as it monitors more units when a process is less capable in its effort to reduce the risk of allowing a defect to occur. It is, also, more sensitive than PC at detecting mean shifts in a process.

SUPA is a nonparametric methodology and should, therefore, be robust against processes with non-Gaussian distributions. This assertion is examined in the following section.

## 5.6 Non-Parametric Analysis

### 5.6.1 Introduction

The analysis performed in section 5.5 determined that SUPA was the most effective non-parametric control method and SB$\bar{X}R$ was the most effective parametric control method, for set-up dominant processes. In this section these two control methods are assessed further by examining their performance against processes with unknown, non-Gaussian distributions. The statistical distributions modeled were (a) Uniform, (b) skewed Triangle and (c) mixed Uniform and skewed Triangle distributions. In order to make the analyses comparable, the variances of the three distributions used were made equal to one another.

The three distributions were selected as they are bilateral distributions, i.e. they can have positive or negative outcomes. This is important, as previous simulation work of Pre-Control by Sinibaldi [127] was criticised in the literature by Shainin [110] for using a $\chi^2$ distribution, which is not bilateral, to test robustness against non-Gaussian distributions. Also, the distributions have probability density functions that are very different, enabling SUPA's non-parametric properties to be examined, as well as testing SB$\bar{X}R$ with processes that are not Gaussian. These three process cases, had an output ($X$) which is expressed mathematically, using the upper, $U$, and lower, $L$, specification limits, the current process mean, $\mu(k)$, and the capability defined at the start of the simulation, $C_p$, by:

(a) Uniform,

$$X \sim U(a_U, b_U); \tag{5.7}$$

where,

$$a_U = \mu(k) - \sqrt{12\left(\frac{U-L}{6C_p}\right)^2}. \tag{5.8}$$

$$b_U = \mu(k) + \sqrt{12\left(\frac{U-L}{6C_p}\right)^2}. \tag{5.9}$$

(b) Skewed Triangle,

$$X \sim Tri(a_T, b_T, c_T); \tag{5.10}$$

where,

$$a_T = 3\mu(k) - b_T - c_T, \tag{5.11}$$

$$b_T = \mu(k) + \left(\frac{U-L}{6C_p}\right), \tag{5.12}$$

$$c_T = \sqrt{((3b(\mu(k)))^2) - (12(3(\mu(k))^2 - 9b\mu(k) + 3b)^2 - 18\left(\frac{U-L}{6C_p}\right)^2))}. \tag{5.13}$$

(c) Mixed Uniform and skewed Triangle,

$$X \sim \begin{cases} Tri(a_T, b_T, c_T) & \text{for} \quad \mu(k) < T - 0.3(T-L) \\ U(a_U, b_U) & \text{for} \quad T - 0.3(T-L) < \mu(k) < T + 0.3(U-T) \\ Tri(2\mu(k) - a_T, 2\mu(k) - b_T, 2\mu(k) - c_T) & \text{for} \quad \mu(k) > T + 0.3(U-T) \end{cases} \tag{5.14}$$

As SB$\bar{X}R$ and SUPA have different philosophies, setting SB$\bar{X}R$s control limits based on a historical process $C_p = 2.0$ and setting SUPAs control limits based on a required $C_p = 2.0$, allowed equivalent analyses. In order to prevent deterministic effects of the simulation's pseudo-random number generator disrupting the outcome, each setting was replicated 1,000 times. In these experiments, as in the previous parametric analysis, if a false adjustment signal was made, the process was adjusted and continued until the control procedure determined the process as either valid or incapable.

As in the previous section, this analysis examines the two events that lead to a control chart signal: a) when a process is on-target but not capable; b) when a process is capable but off-target. These two situations were applied to the three process distribution cases: (a) Uniform; (b) skewed Triangle; (c) mixed Uniform and skewed Triangle.

### 5.6.2 On-target, not capable

In these cases, the process had (a) Uniform, (b) skewed Triangle and (c) mixed Uniform and skewed Triangle distributions, whose means ($\mu(k = 0)$) were on-target with respect to the nominal specification ($T = 100$). However, the distributions variances were increased in each simulation, to test how effective the respective control methods were at detecting that the process was not capable. As $C_p$ is inversely proportional to the standard deviation, when the process variation is increased, $C_p$ is reduced.

Results for the probability of qualifying, despite adjustment, against the process $C_p$, are given in figure 5.14 (a;b;c). These results demonstrate that if $C_p = 2.0$, both SUPA and SB$\bar{X}R$ validate the process in excess of 98% of occasions, despite the underlying statistical distribution of the process. Also, it is shown in Figure 5.14 that SUPA is more powerful as a control method than SB$\bar{X}R$. This is shown with the same type of separation in performance seen in the previous parametric analysis in these nonparametric cases (a;b;c) and was particularly pronounced in the Uniform case (figure 5.14(a)). This is an important point, since it indicates that SUPA rejected more invalid processes than the SB$\bar{X}R$ method.

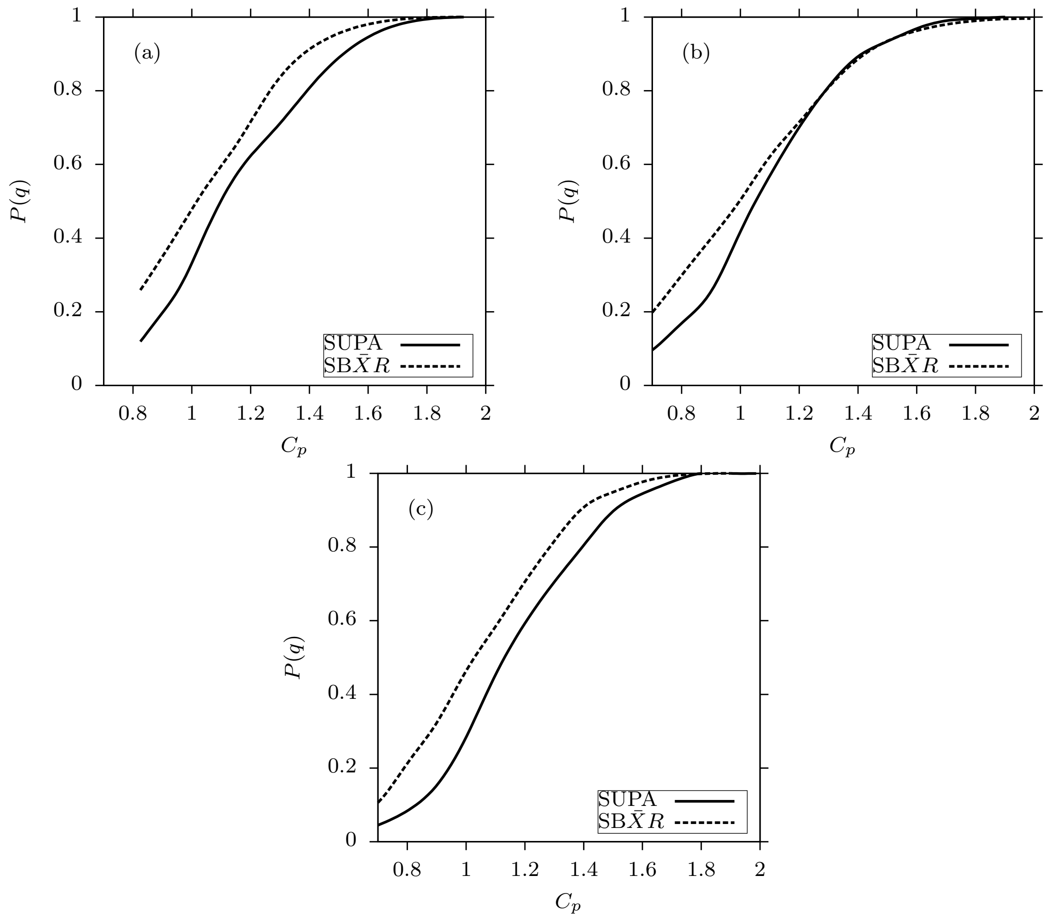


Figure 5.14: Operating curves of decreasing $C_p$ against probability of qualifying ($P(q)$) for (a) Uniform, (b) skewed Triangle and (c) mixed distributions.

The number of units sampled, by the control methods, before making a final decision was also assessed, Figure 5.15(a;b;c). The results show that SB$\bar{X}R$ consistently uses approximately six samples before making a final decision whether the process is valid or not for all distribution cases. However, as in the parametric case, SUPA uses a small number of units to make a final decision when the $C_p$ is close to 2.0, but uses increasingly more units as the $C_p$ decreases. For example, SUPA uses a maximum of: 18 samples in the uniform case (figure 5.15(a)); 14 samples in the skewed triangular case (figure 5.15(b)); 16 samples in the mixed case (figure 5.15(c)).

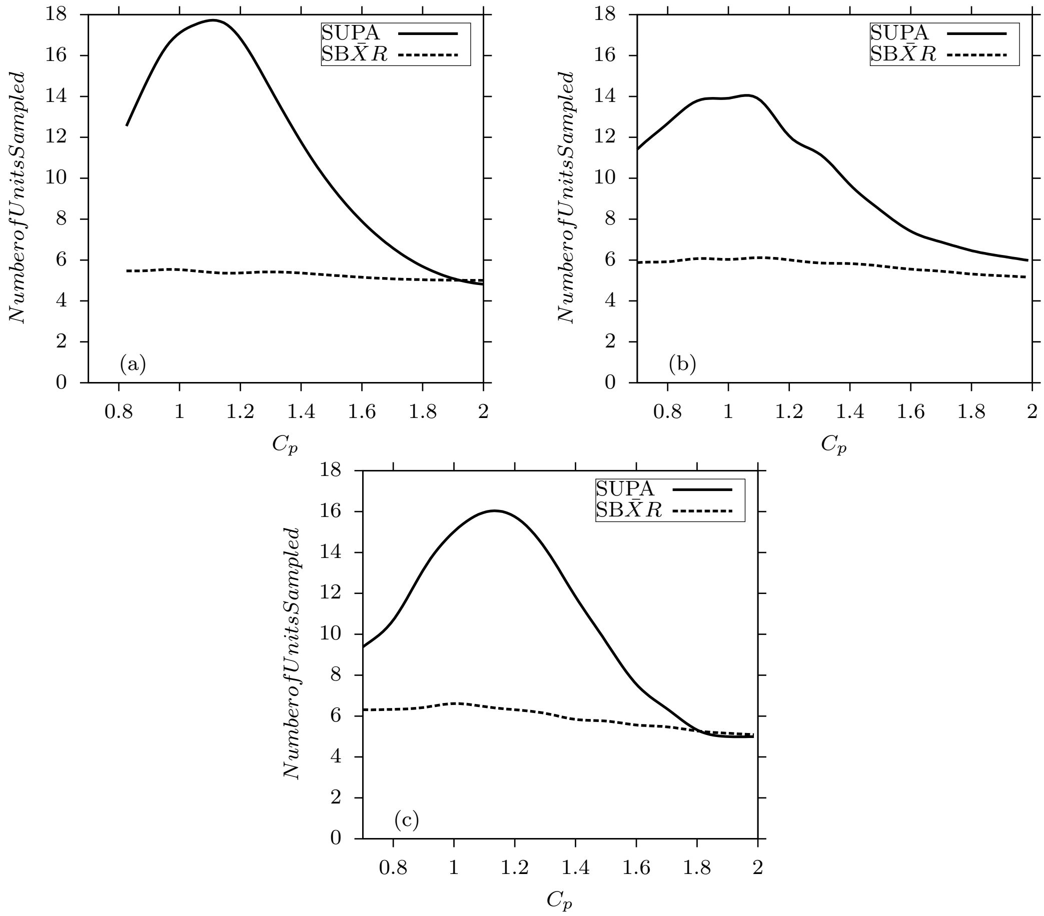


Figure 5.15: Operating curves of decreasing $C_p$ against number of samples for (a) Uniform, (b) skewed Triangle and (c) mixed distributions.

As in the previous parametric analysis, this nonparametric analysis has shown that both SUPA and SB$\bar{X}R$ are both suitable tools for low-volume process control in terms of identifying incapable processes. The performance of SB$\bar{X}R$ in this nonparametric analysis is particularly surprising, given that it is a tool based on parametric assumptions. Again, this analysis has shown that SB$\bar{X}R$ is more efficient in terms of the number of units sampled, but SUPA gives better protection against accepting invalid processes. The next section of this nonparametric analysis assesses SUPA's and SB$\bar{X}R$'s ability to identify and off-target process with a underlying non-Gaussian distribution.

### 5.6.3 Off-target, capable

The second situation was for a capable process, i.e. had a Cp=2.0, but with an initial process mean not on-target ($\mu(k=0) \neq T$). As with the previous test, in this section, the process had (a) Uniform, (b) skewed Triangle and (c) mixed distributions. At the start of each simulation run, the process's starting mean ($\mu(k=0)$) was moved further off-target. The standard deviation, $\sigma$, of the process was calculated, given a $C_p = 2.0$, using equation (3.7). The deviation from the design target was measured as a coefficient, $\varrho$, of $\sigma$, such that:

$$\mu(k=0) = T + \varrho\sigma. \tag{5.15}$$

Figure 5.16(a;b;c) highlights the final capability with respect to process accuracy and precision ($C_{pk}$), plotted against $\mu(k=0)$'s starting position. This shows that:

- in the Uniform case (Figure 5.16(a)), profiles were closer in performance;
- in the skewed Triangle case (Figure 5.16(b)), the dip in performance was $C_{pk} \approx 1.6$ at $2.5\sigma$ for SUPA and $C_{pk} \approx 1.8$ at $1.5\sigma$ for SB$\bar{X}R$;
- in the mixed case (Figure 5.16(c)), there was a dip in performance of $C_{pk} \approx 1.6$ for SUPA and $C_{pk} \approx 1.8$ for SB$\bar{X}R$.

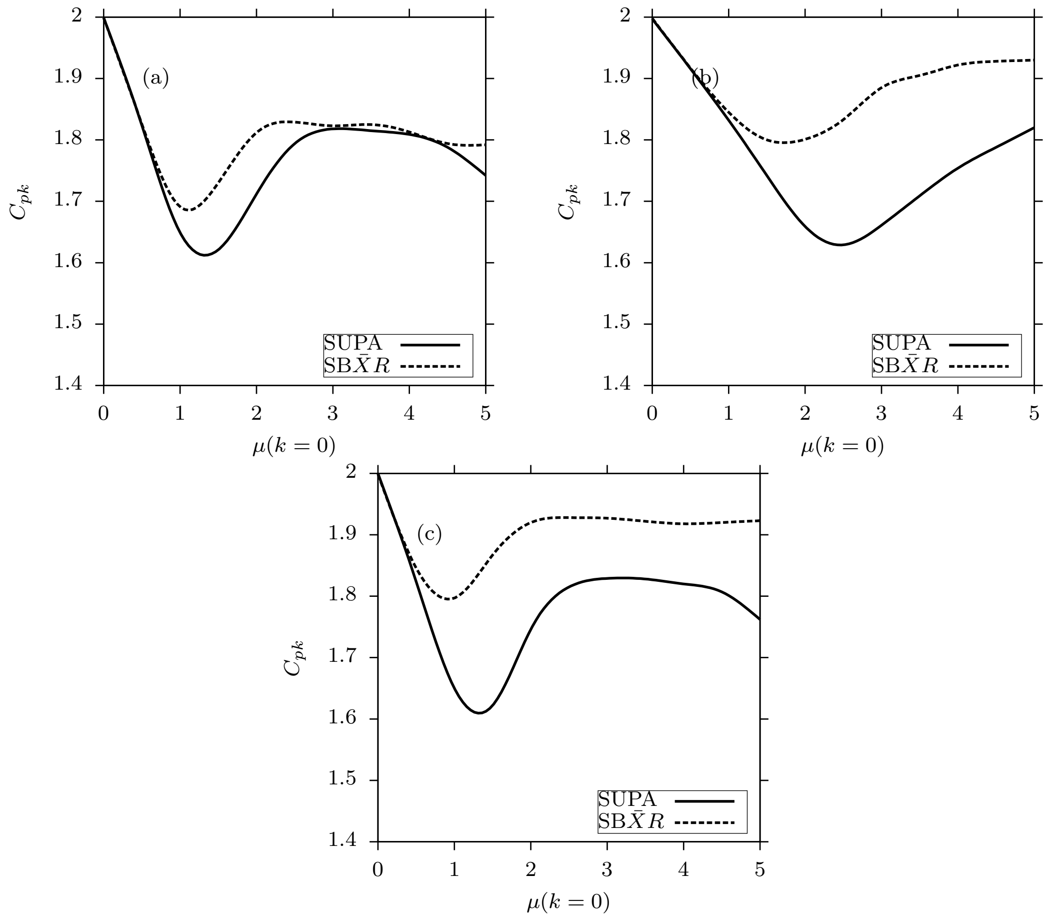


Figure 5.16: Operating curves of final $C_{pk}$ against process mean start positions $\mu(k = 0)$ as a factor of $\sigma$, when $C_{pk} = 2.0$, for (a) Uniform, (b) skewed Triangle and (c) mixed distributions.

Figure 5.17(a;b;c) illustrates the number of adjustments made before a final decision, plotted against $\mu(k = 0)$'s starting position. It can be seen in all cases (figures 5.17(a;b;c)), that both control methods had performances in a similar range. Once $\mu(k = 0)$ is more than $1\sigma$ away from the $T$ in the Uniform and mixed distribution cases and more than $2.5\sigma$ away from $T$ in the skewed Triangle case, they will make $1 - 1.5$ adjustments to the process on average. The closeness in profiles in both the final $C_{pk}$ (Figure 5.16) and number of adjustments made (Figure 5.17), shows that both methods respond and make similar corrections to the system.

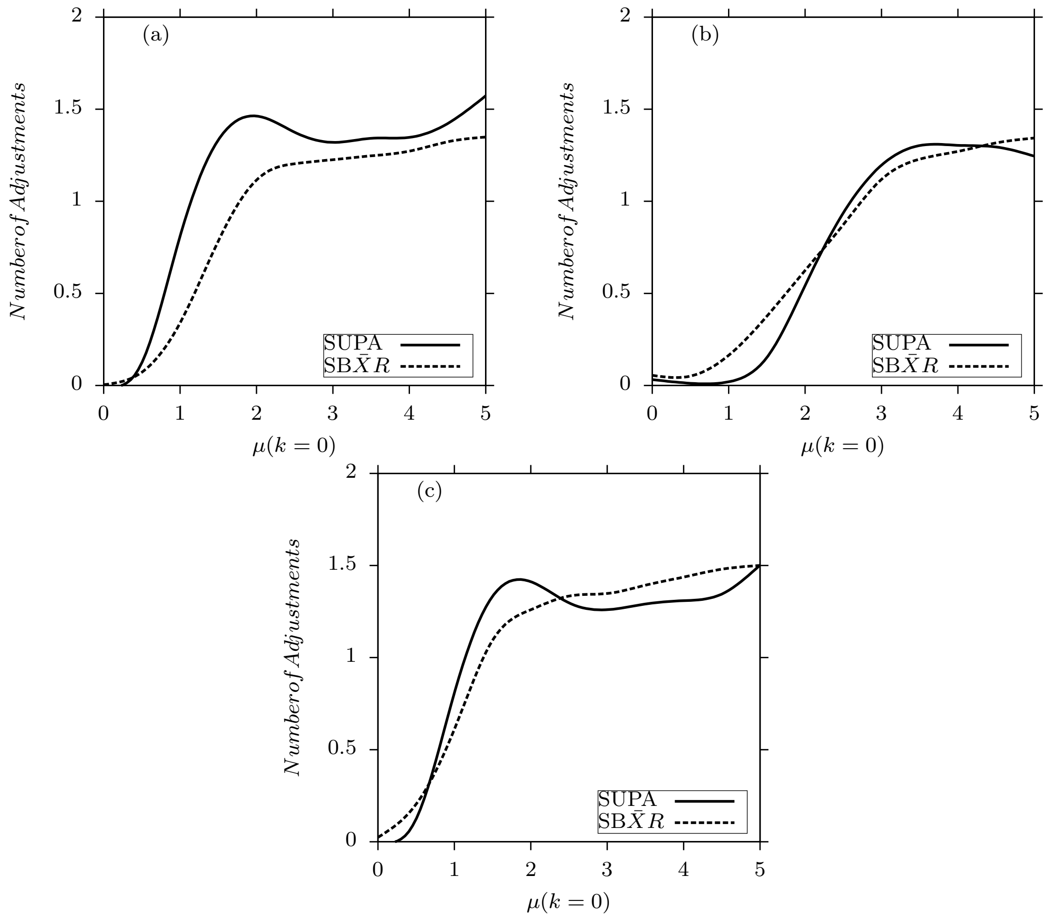


Figure 5.17: Operating curves of increasing $\mu(k = 0)$ as a factor of $\sigma$, against number of adjustments for (a) Uniform, (b) skewed Triangle and (c) mixed distributions.

Figure 5.18 illustrates the number of units sampled before an adjustment is made. It was shown in these results that the SB$\bar{X}R$ method takes less samples, before making an adjustment, than SUPA. SB$\bar{X}R$ makes a decision to adjust a process within five samples or else validate the process, whereas, SUPA takes as many as necessary to come to a conclusion. It is also worth noting, that the extreme situations shown in Figure 5.18 of SUPA taking more than six units to make an adjustment, typically when $\mu_0$ was less than $1\sigma$ off-target, occurs infrequently. This result only shows the number of samples to make an adjustment in the runs when an adjustment occurs. If we take this in context with the results in Figure 5.17, when $\mu_0$ was less than $1\sigma$ not all runs were adjusted and were validated instead. This leads to the 'dips' in performance seen in the $C_{pk}$ values shown in Figure 5.16, but prevents excessive sampling.

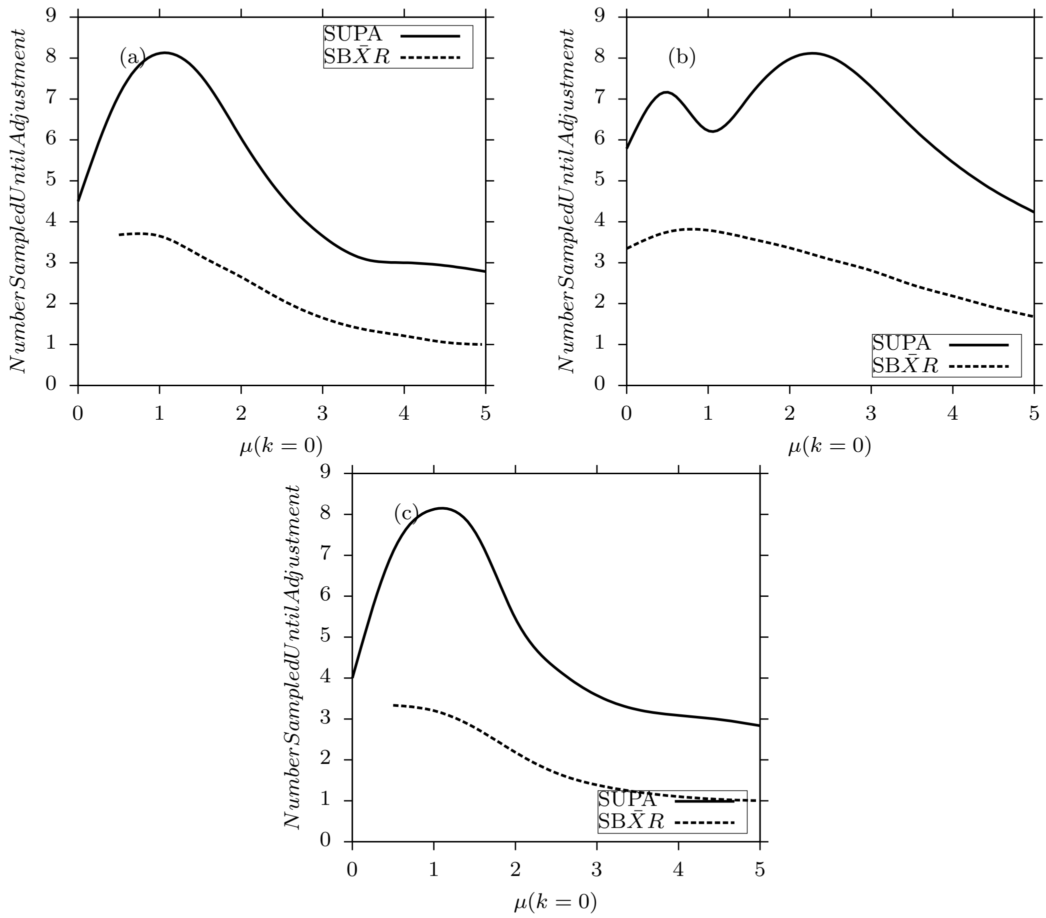


Figure 5.18: Operating curves of increasing $\mu(k = 0)$ as a factor of $\sigma$, against number of sampled units before an adjustment for (a) Uniform, (b) skewed Triangle and (c) mixed distributions.

Figure 5.19 illustrates the number of units required to make a final decision. It was noted that the typical number of units sampled was in the range 5-8 for SB$\bar{X}R$ and 6-14 for SUPA. This difference is in keeping with the results described in Figure 5.18. Both SB$\bar{X}R$ and SUPA need at least five units to validate, this result is, therefore, created by the difference in the number of sampled units before an adjustment is made.

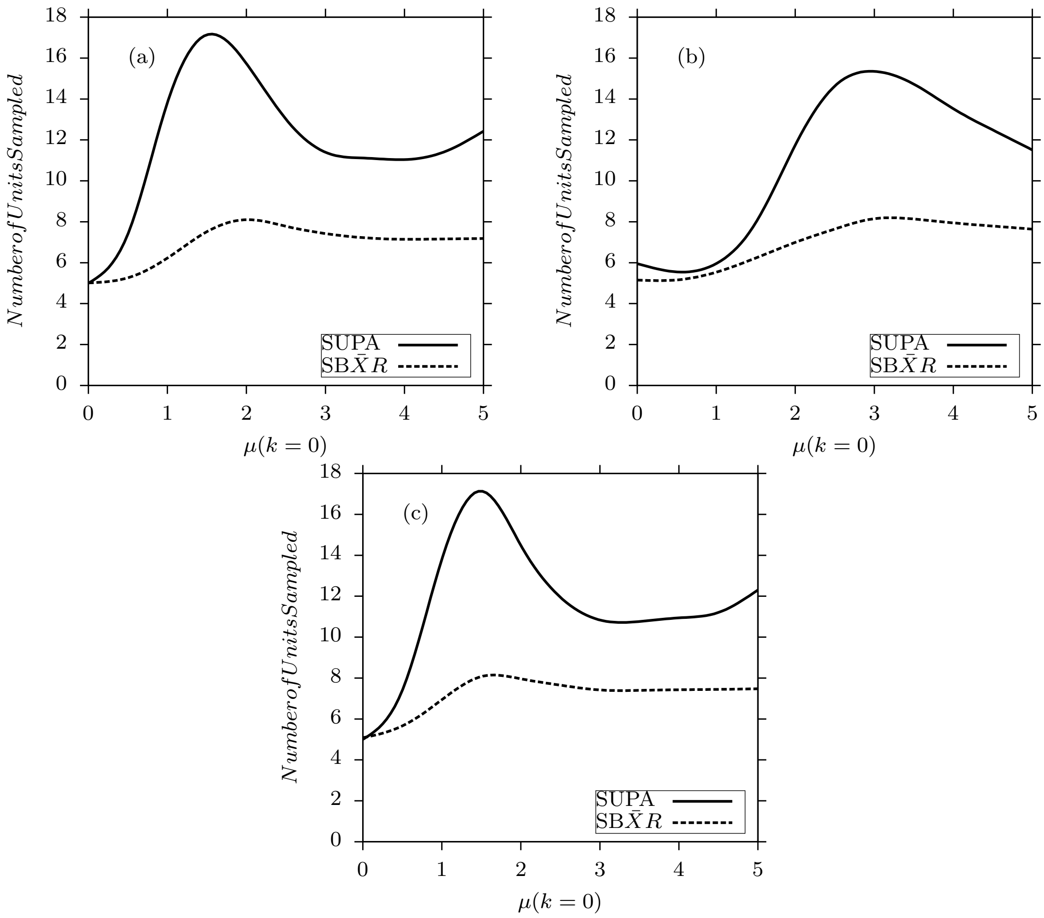


Figure 5.19: Operating curves of increasing $\mu(k = 0)$ as a factor of $\sigma$, against the total number of samples for (a) Uniform, (b) skewed Triangle and (c) mixed distributions.

Overall, it can be seen that the performance of the methods was similar. SB$\bar{X}R$ was faster at making a decision and slightly better at maintaining a process $C_{pk}$ as close to its optimum. However, SUPA was more powerful at detecting a process that was incapable and in this instance the additional sampling was useful.

### 5.6.4 Summary

This section compared the performance of two set-up dominant process control methods: SB$\bar{X}R$ and SUPA, against processes with non-Gaussian distributions. The former provides a statistical process control procedure, which maintains the statistical stability of a process using capability limits. Whereas, the latter is a non-parametric approach to defect prevention, with control limits and tests based on specification and derived using probability theory. The method's effectiveness with respect to: (a) detecting an incapable process; (b) adjusting an off-target process, were studied. To investigate this, the methods were used to control a simulated process with three different statistical distributions.

## 5.7 Summary

In section 5.1 the need for a process control tool to monitor set-up dominant processes was identified. Three currently available approaches: Pre-Control, ACC and SB$\bar{X}R$, were highlighted. However, each had their own theoretical and practical limitations; as a result, it is common for operators of setup dominant processes to use a rule-of-thumb approach to process control.

In section 5.2, this issue of monitoring set-up dominant processes was then observed in a case study of a manufacturer and global supplier of quality metal printing machinery. During this case study the initial observations, match those in the literature, that rule-of-thumb methods are being used to control the setup dominant processes within the company. Management perceived classical SPC approaches as not relevant for control of their processes; operators saw classic SPC as an abstract approach, which did not link control limits and tolerance limits.

In order to improve the situation, Pre-Control was trialled as a potential method of process control. Pre-Control has had limited use in these setup dominant situations, but its phase 1 can validate a new process without having prior statistical knowledge. It was selected as it provided operators within easy to grasp traffic-light chart, linking tolerance limits to control limits. Although, the paper-based system used in the trial was convoluted, the operators appreciated the easy to interpret traffic light chart and system of rules to identify incapable processes. The main problem highlighted in the trial, was up Pre-Control was a very rigid system and did not have a mechanism to control processes required different levels of capability.

To address this issue, a new method of set-up dominant process control, called Set-Up Process Algorithm (SUPA), was developed and described in section 5.3. This approach was largely based on the Pre-Control methodology, due to being nonparametric and an easy to follow system of traffic-light chart and rules.

To fully examine the potential of SUPA and to compare it with other set-up dominant process control methods, a stochastic discrete-event simulation model was developed as a test bed. This model was described in section 5.4 and provided a means of assessing process control tool performance features that cannot be calculated analytically. The results derived from these models, therefore, give a more complete picture to evaluate the effectiveness of each tool. The situation experiments were then broken into two sections: firstly, a parametric analysis in section 5.5; secondly, a nonparametric analysis in section 5.6.

In the parametric analysis, SUPA, Pre-Control, ACC and SB$\bar{X}R$ were compared to one another against a set-up dominance process with a Gaussian distribution, in two situations: a) when a process was on-target but contained excessive variation; b) when a process had a fixed amount of variation but was off-target. The results from this study, showed that SUPA and SB$\bar{X}R$ were overall the best performing process control tools for a set-up dominant process with a Gaussian distribution.

In the nonparametric analysis, SUPA and SB$\bar{X}R$ were compared against one another against a set-up dominant process with three different non-Gaussian distributions, namely: a) uniform; b) skewed triangle; c) mixed uniform and skewed triangle. It was noted, that the SB$\bar{X}R$ control method performed well against different statistical distributions, despite it being specifically designed to operate with Gaussian processes. However, the performance results between SUPA and SB$\bar{X}R$ largely followed the same pattern as the parametric analysis.

A consideration from the simulation results is the number of units required to make a final valid or not valid decision. It was shown in all cases that the SUPA method used more samples than SB$\bar{X}R$. When the process was capable and on-target it was shown that the difference between the two methods is marginal. However, as a process becomes less capable, or starts further from the target, the SUPA method requires many more samples to make a final decision. This can be seen as a useful feature, for the situation where the process is not capable. The SUPA method is more powerful in this situation, as a result of additional sampling when processes become incapable. Also, if the process is not capable it produces more defects; therefore, by collecting more samples the chances of detecting a defective units is increased. The high sample numbers collected by SUPA in the second situation, where a process is off-target, is less desirable. It is preferable in this situation that an adjustment is made and then validated as quickly as possible.

A practical consideration, which cannot be explored by simulation, is how easy it is for the end user to implement the methods. These methods are typically used by machine operators, who are unlikely to have a background in statistics. Therefore, they are likely to find a method which uses control limits linked to design tolerance easier to utilise, such as SUPA, than ones which use statistical limits, such asSB$\bar{X}R$.

To extend this work further, the following chapter addresses the situation where set-up dominant processes are also multivariate in nature, i.e. they have multiple correlated CtQs.

# Chapter 6

# Multivariate SUPA



## 6.1 Introduction

In the previous chapter, a new traffic-light control chart for setup dominant processes was introduced known as SUPA. It was shown through simulation analysis that SUPA was an effective control tool for monitoring processes producing products with a small number of CtQ features. Also, SUPA required independence between CtQs and the process's adjustment or re-centring parameters, for example, in a machine tool this would be the machine tools offsets.

In chapter 1, it was outlined that the AMTs that typically operate in a set-up dominant environment producing parts with ever increasing complexity. An example from the aerospace industry is the production of aerofoil parts for jet engines. In this example, it is common for in-excess of 30 CtQ features to be machined on the part by a single multi-axis Computer Numerical Control (CNC) machining centre process. This would require the machine tool operator to manage over 30 separate SUPA charts for a single part and process combination. An industrial case study of this type of issue is outlined and examined in section 6.2.

It is clear there is a need to extend univariate approaches to process control, such as SUPA, into a multivariate context. In section 3.4, the literature on multivariate process control was reviewed. As with the literature on univariate process control, there has not been extensive work in the field of multivariate process control for setup dominant processes. Most of the multivariate process control methods available are extensions of classic univariate

SPC techniques. This means they require large sampling during the start-up period to derive control limits based on the process following an approximately Gaussian multivariate distribution. There were two papers reviewed [115, 116], that attempted to extend pre-control into a multivariate context. The issue with these approaches was that they were not truly nonparametric. Instead, they offered a version of multivariate stop-light control, whereby, the control boundaries were calculated using statistical limits and historical data. In response to this, section 6.3 extends the SUPA method into a multivariate approach. This method, known as Multivariate Set-Up Process Algorithm (mSUPA), provides an operator with the same traffic-light based rules of SUPA that are nonparametric; hence, do not require prior process knowledge. The implementation of this new mSUPA method is then discussed in section 6.4.

In section 6.5, the performance of the mSUPA is assessed using a discrete event simulation model. The discrete-event simulation model used is the same as described in the previous chapter, in section 5.4, has been extended to produce products with multiple CtQs. The findings of this chapter are concluded in section 6.6.

## 6.2 Case Study

### 6.2.1 Introduction

This case study was conducted at manufacturing facility of a world leading producer of aero engines. The study consisted of a site visit to the manufacturing facility on 23/10/13 and follow-up analysis of production data. The manufacturing facility mainly produced aerofoil parts, known as vanes, of varying types and sizes. These vanes would then go into a final assembly producing stators. The stators are then used in different sections of a jet engine; for example, a high-pressure stator would be used for the engine's hot section or a compressor stator for the engine's cold section. The function of the stator, ultimately, defines the specification of the vanes CtQ features.

In this case study, a single manufacturing process is followed. It consists of a single multi-axis CNC machining centre. Typical production volumes in this process are between 40-100 parts. After a batch of parts has been made, the process is set up to manufacture a different component. This is a typical set-up dominant process, because the production volume is too small to calculate control limits for classical SPC techniques. The operators of this process have access to a CMM inspection station to check a part's CtQs for dimensional conformity against the CAD model. A unique feature of this manufacturing process is that the machining centre has four heads; which means that, any adjustment to a machine tool offset will not be seen in the part CtQ for three units. It is therefore critical for the company to have an effective process control methodology in place, that can detect a drift in the process before it is out-of-tolerance. If a part as measured out-of-tolerance, even if an immediate change is made to the process, the next three parts are likely to be also out-of-tolerance.

Also, this study focuses on the production of a single part; a standard vane, for a stage II high-pressure stator. This part has 26 CtQ features machined during this process step. A drawing of this part and the CtQ features is shown in figure 6.1. The next section outlines process control practices observed during the site visit on 23/10/13.

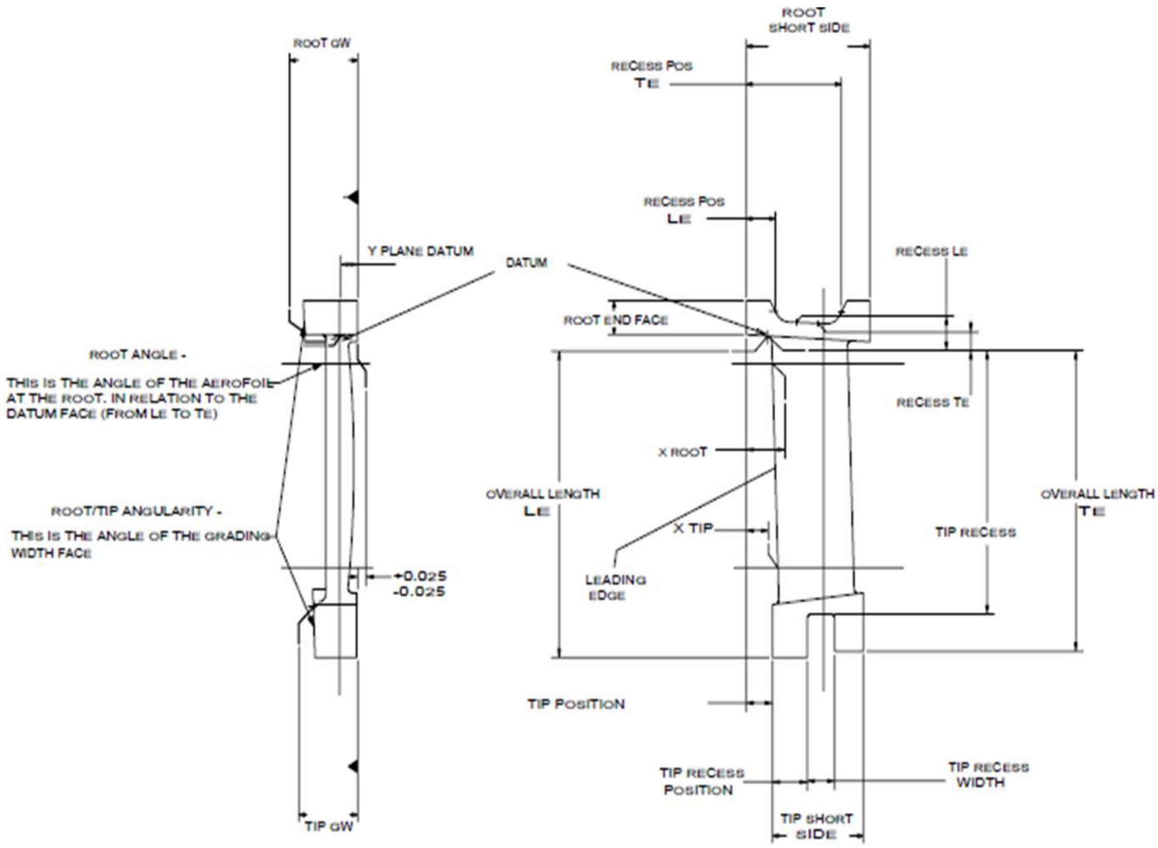


Figure 6.1: Drawing of the standard vane for a stage II high-pressure stator.

### 6.2.2 Observed Practices

At this stage current practices were observed, with a particular focus on the process control methodology employed. As in the previous case study, set-up time of the process was significant compared to the processing time of a single part. It was, therefore, critical that if a batch size of 100 parts was required, operators were able to produce 100 in-tolerance parts. In order to make the facility's production more lean and to reduce work in process, the company had moved away from *' over-producing'* parts to compensate for any potential defects. However, this puts pressure on the operator to produce the required number of in-tolerance parts; if defective parts went undetected and replacements had to be made, this would require the process to be set-up again for a batch size of one part. This in turn, reduces the overall manufacturing capacity of that particular machining centre, due to the extra time spent in a set-up mode and no part produced.

The company wants to minimise the number of parts that were inspected. Employing a variant of SPC was seen as a potential methodology to give them confidence in reducing the number of samples inspected. Prior to the site visit, the company had completed a review of potential SPC practices to employ. This review included input from leading external consultants in the field of SPC. The findings in this review concluded that there was no currently available SPC method suitable for their purpose.

This led to the current situation where 100% inspection is employed. The measurements collected from the CMM were automatically collected using *Mitutoyo MeasurLink*. Using this software, operators were able to automatically plot their measurements onto Pre-Control charts; although they did not employ any formal Pre-Control rules. Operators were merely using the Pre-Control chart as a guide, trying to set up their processes within the green band and not just within tolerance. Although, Pre-Control charts could be plotted through the software, the standard rules to identify out-of-control processes were not built in. Management found that there was an overall improvement in productivity as a result of using these Pre-Control charts as a guide, but it had not succeeded in reducing the number of inspected parts. In fact, employing this Pre-Control approach had increased inspection time, as operators would have to review 26 charts associated with each CtQ. This was an issue the company wished to address and is a situation where a multivariate approach to analysis would be suitable.

The company had two fundamental issues with employing a full Pre-Control implementation. Firstly, there were certain parts and CtQs that would require confidence that the process capability $C_{pk} \geqslant 1.33$, in order to reduce the current 100% inspection regime. This is not achievable with standard Pre-Control. Secondly, there were situations where to keep one CtQ near its target, another CtQ would have two run in a yellow zone on a Pre-Control chart. This situation occurs due to a combination of tool wear and correlation of two CtQs to a single machine tool offset, i.e. changing this offset has an effect on both CtQs. The operator, therefore, has to make a decision as to what is the optimum position to run the process. It may be that one CtQ is less critical to the overall in-service performance of the part and this is allowed to be manufactured closer to its tolerance limit. The result being that the other CtQ can be manufactured closer to its target value.

### 6.2.3 Discussion

This site visit had positive discoveries. Chiefly, that operators were not just trying to steer their processes to be inside the tolerances, but were using a Pre-Control chart to steer the process into the middle 50% of the tolerance band. However, no Pre-Control rule sets were used to give operators confidence that their processes were capable. Therefore, production could not move away from their current 100% inspection regime. There was also concerns over the time added by monitoring multiple univariate Pre-Control charts for each part type produced and the need to have confidence that certain CtQs had a high capability. The remainder of this chapter addresses these issues by introducing a new multivariate version of

SUPA, a control tool developed for these set-up dominant applications.

A further concern, was the ability to optimise the process. Currently this optimisation is performed by the machine tool operators. Chapter 7 looks into methods of automating this optimisation process in conjunction with a process control tool, such as mSUPA.

## 6.3 Multivariate SUPA

The SUPA method, described in chapter 5, provides a machine operator with a simple chart and rule set to statistically diagnose when a process is off-target. The chart is based around the tolerance of the monitored CtQ design feature and a traffic-light scheme where the central region around the design target is designated as the green zone. The regions which are between the green zone and the tolerance limits are the yellow zones. The regions outside the tolerance limits are the red zones. The size of the green zone is determined by the minimum $C_p$ required from the process, as per figure 5.2.

If the green zone percentage is used from figure 5.2, this means that the probability of a single part falling into the green zone is $P(g) = 0.94$ if it conforms to the minimum required $C_p$. Consecutive parts are sampled and their measured CtQ design features are categorized as green, yellow or red. If a sampled part is red it signals that the process is off-target. Two consecutive parts in the same yellow zone signal an off-target process. Five consecutive green parts demonstrate the process is capable with 98% confidence and is allowed to continue without further checks. These rules are summarised in table 5.2.

In order to perform SUPA in the multivariate case (mSUPA), in a manner that maintains a nonparametric approach a series of sampling rules needs to be established. In order to maintain consistency between univariate and multivariate SUPA, the rules established in table 5.2 will be used in the multivariate context. Therefore, this requires that consecutive parts are sampled from a process after set-up. The CtQ features of these parts are measured and recorded as the CtQ vector, $x$. If a feature of the CtQ vector is outside its design tolerance, the part is scrapped.

Consider the simplified case of a part with two CtQ design features, i.e. $x = [x_1, x_2]^T$. If $x_1$ and $x_2$ have the same specified design tolerances of $U = 250$ and $L = 50$, the tolerance boundary can be represented as a box, as in Figure 6.2. Taking the philosophy of linking a red zone to design tolerance from SUPA, this box can be used in mSUPA as a boundary between the red and yellow zones. Let two measured parts $k = 1$ and $k = 2$ be collected with CtQ design vectors of $x(k = 1) = x(1) = [200, 100]^T$ and $x(k = 2) = x(2) = [40, 200]^T$. These points are plotted on figure 6.2, where it is shown there that $x(1)$ is within and $x(2)$ is outside the design tolerance.

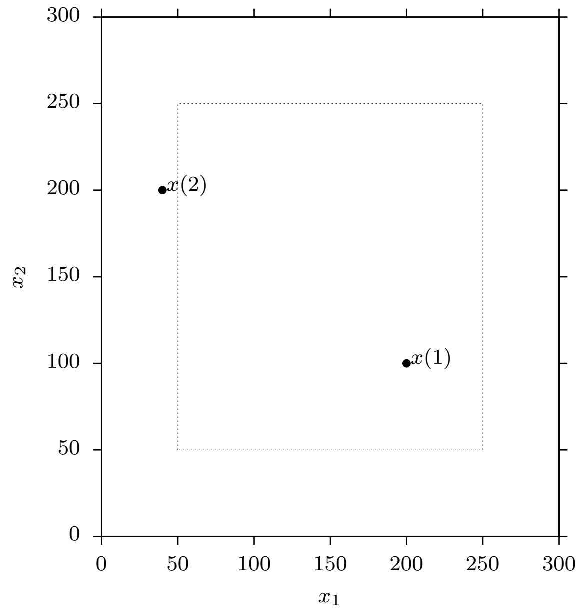


Figure 6.2: Tolerance boundary of $x$ and positions of CtQ design vectors $x(1)$ and $x(2)$.

Although this information tells a user if a part is in or out-of-tolerance, it does not give any indication of how close a part is to the design target. However, to formulate a green zone, and therefore an mSUPA chart, a target $C_p$ value for each CtQ design feature needs to be defined. Rearranging the $C_p$ formula the desired standard deviation is:

$$\sigma = \frac{U - L}{6C_p}. \tag{6.1}$$

For an $x$ with $n$ CtQ design features, the minimum variation in each $x_i$, $i = 1, \ldots, n$ can be represented by $\sigma_i^2$. This allows the definition of the target covariance matrix, $S$ as:

$$S = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \sigma_n^2 \end{bmatrix}. \tag{6.2}$$

The target covariance matrix, $S$ only contains diagonal elements. This reflects the fact that no assumptions are made about correlations between CtQ design features. With $S$, the multivariate chart in Figure 6.2 is refined by using this as a scale of maximum variation acceptable in the process. $S$ is used with the measured CtQ design vector, $x$, and the process target vector, $T$, to calculate the Mahalanobis distance [128], between $x$ and $T$ as follows:

$$(x - T)^T S^{-1}(x - T) < H^2 \tag{6.3}$$

where $H$ is a pre-selected constant. This allows the definition of a multidimensional green zone, which is the set of those points that have a Mahalanobis distance less than $H^2$ from $T$. The left hand side of equation (6.3) has the property of following a $\chi^2_{n,\varepsilon}$ distribution, where

$n$ is the degrees of freedom, which is equal to number of CtQ design features and $\varepsilon$ is the probability of a sample from a population that is on-target falling outside the green zone.

In the case of univariate SUPA the green zone is defined so that a part produced by an on-target process has a minimum probability of falling in the green zone of $P(g) = 0.94$. Hence, extending this to the multivariate case results in $\varepsilon = 0.06$. This results in the sphere shown in figure 6.3. Using this chart, a decision about whether a process is off-target or not, is still made by following the SUPA rules of table 5.2.

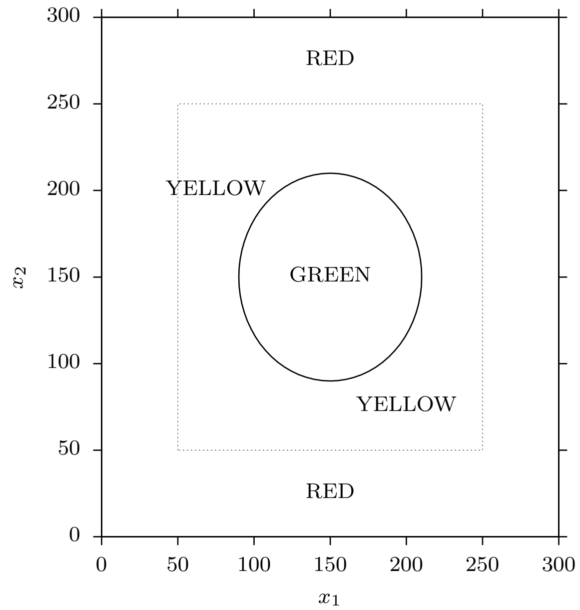


Figure 6.3: 2-dimension mSUPA chart showing green, yellow and red zones.

A significant issue with all multivariate approaches to process control, and mSUPA is no exception, is how to present information. The next section, addresses how to implement an mSUPA process control system.

## 6.4 Implementation

A key point in any successful process control method is how it is implemented. As was shown in the univariate Pre-Control case study at a manufacturer of metal printing presses in section 5.2, the management of control charts needed to be perceived as effortless to ensure widespread uptake. In the case study, a paper-based method of implementing univariate Pre-Control was used for a trial purpose. Although this paper-based method provided useful data to the operator in the trial, it was perceived as a hassle to complete which would limit its widespread uptake.

This issue has been overcome with the implementation of classic SPC techniques in high-volume manufacturing applications. Software packages, such as *Mitutoyo MeasurLink* [129], *InfinityQS ProFicient* [130],*Q-DAS procella* [131], to name a few, offer automated solutions for real-time data collection and SPC. It was shown in the manufacturer of aerofoil parts case study in section 6.2, that these systems offer a convenient solution to collect and visualise data. However, the SPC solutions built into the software packages are not statistically robust against set-up dominant processes and do not offer an effective way to communicate multivariate data.

A solution to providing a process control methodology that was robust against set-up dominant processes, would be to offer mSUPA through a real-time data collection and process control software package. The critical issue then would be how to present multivariate data available through mSUPA. As the number of CtQ features on a single part increases, with increasingly complex designs, successful implementation does not just rest on an easy-to-use data collection system but also requires a means of presenting data to an operator in a way that is digestible. If for example, a part with 25 CtQ features was to be monitored, either a single plot with 25 axes would be needed or 25 individual projection plots are required. The former would be impossible to visualise given the stereoscopic 3-D vision of the human eye. The latter has already been shown to be time-consuming, as it would be a very similar process to the management of multiple Pre-Control charts that is currently contacted by the company in the aerofoil parts case study.

However, visualising data graphically would miss a key feature of the mSUPA method. That is a simple traffic-light could be presented to the operator. Once CtQ tolerances and mSUPA zones have been specified, it is computationally quick to identify which zone a measured part falls into. Presenting an operator with a green, yellow or red light gives them very quick information about the current global position of the process. If a red or two consecutive yellows are signalled individual 2-D projection plots can then be presented to the operator, to assist in identifying what corrective action is needed.

The next section uses the discrete-event simulation model described in section 5.4 to test the performance of this new mSUPA method.

## 6.5 Detection of Off-Target Processes

### 6.5.1 Introduction

In this section the mSUPA approach is tested and benchmarked against the first-off approach using a discrete-event simulation model. The model used is the same as described in section 5.4, but it now produces products with multiple CtQ features. For the purposes of presenting results that can be visualised, this analysis looks at a simulated process that produces a product with two CtQ features ($x$), therefore:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{6.4}$$

These products are produced by a process that has a multivariate normal distribution. This means that $x$ is distributed by:

$$x \sim N(\mu, \Sigma), \tag{6.5}$$

where, $\mu$ is the vector mean and $\Sigma$ is the covariance matrix of the CtQ features. In the simulations, the positioning of $\mu$ is changed to assess the effectiveness of mSUPA at detecting whether a process is making CtQs near their design target. In these examples, both $x_1$ and $x_2$ have the same specified design tolerances $U = 250$, $L = 50$ and a design target $T = 150$. The covariance matrix, $\Sigma$ is adjusted to provide to separate simulation cases: the first in section 6.5.2, looks at the situation where there is no correlation between the two CtQ features; the second in section 6.5.3, looks at the situation where there is a strong correlation between the two CtQ features.

In this analysis, either the mSUPA or the first-off method is being used to identify whether $\mu$ is sufficiently close enough to $T$ to validate or qualify the process. If $\mu$ and $T$ are not close enough together, a signal is made that an adjustment is needed to the process. The first-off method achieves this by sampling the first unit made then, if $x$ is within its tolerances the process is validated, if $x$ is outside one of its tolerances the process is rejected. For mSUPA a target $C_p$ value for each CtQ needs to be defined, so that a green zone boundary can be calculated. For the purposes of this simulation, both $x_1$ and $x_2$ are expected to have a minimum capability $C_p \geqslant 2.0$. This means that the mSUPA covariance matrix, $S$, is:

$$S = \begin{bmatrix} 277.78 & 0.00 \\ 0.00 & 277.78 \end{bmatrix}. \tag{6.6}$$

Using this value for $S$, a decision can be made as to whether a $x$ value falls in a green zone by solving the inequality in equation 6.3. The value for $H^2$, from equation 6.3, has the property of following a $\chi^2_{n,\alpha}$ distribution. In this case, the degrees of freedom $n = 2$ and the probability that an on-target process produces an $x$ value outside the green zone, $\alpha = 0.06$; therefore, $H^2 = \chi^2_{2,0.06} = 5.627$. The red/yellow zone boundary is encapsulated by $U$ and $L$ for $x$. These green/yellow/red zones can be plotted as in figure 6.3.

In these simulations, the performance of mSUPA to indicate when an adjustment is needed is assessed. This means that adjustments are not made in this series of simulations. The topic of adjusting a multivariate processes is addressed in chapter 7.

### 6.5.2 Two Uncorrelated CtQs

In this series of experiments, the ability of mSUPA and the first-off method to detect an off-target process is tested when there is no correlation between individual CtQ. The process is also producing CtQ features with a capability $C_p = 2.0$. This means that the processes covariance matrix, $\Sigma$, is:

$$\Sigma = \begin{bmatrix} 277.78 & 0.00 \\ 0.00 & 277.78 \end{bmatrix}. \tag{6.7}$$

In the first experiment, the CtQs process means were set equal to the design target, i.e. $\mu = T$. The simulation was then allowed to run uninterrupted, producing 10,000 products. Then the resulting $x$ values were plotted to visualise the distribution of points created by $\Sigma$ value in equation 6.7. These results are displayed in figure 6.4, which shows a dense circular scattering of points within the green zone.

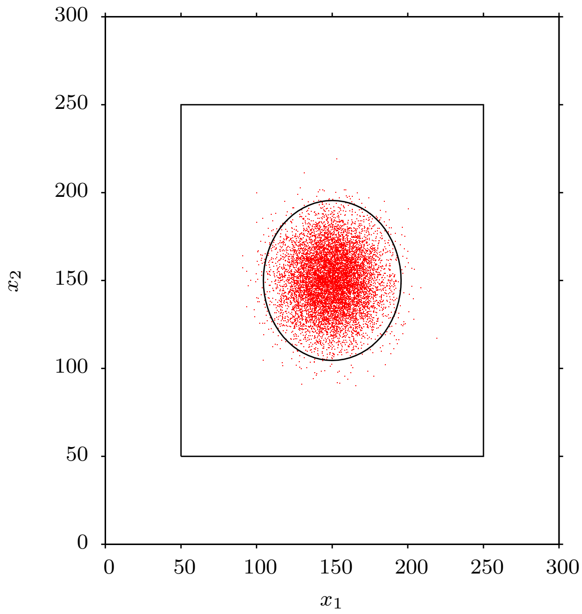


Figure 6.4: Distribution of $x$ values from the uncorrelated simulated process.

This dense circular scattering of points is consistent with two features that are independently distributed. I.e. there is no correlation between the two CtQ features. This confirms that the simulation is producing products with CtQ features as expected.

A second experiment was then run with this simulation; whereby, the starting process mean ($\mu(k = 0)$) was moved progressively further away from the design target. This experiment is performed to demonstrate the effectiveness of mSUPA at identifying and off-target process. Due to the red zone being linked to CtQ design tolerances, this control limit forms a rectangle around the design target. To compensate for the different distances between the central design target and the red zone boundary, this experiment was run twice. In the first run, the position of $\mu(k = 0)$ was moved away from the design target in a direction that crosses the red zone boundary in the shortest path, see figure 6.5(a). In the second run, the position of $\mu(k = 0)$ was moved away from the design target in a direction that crosses the red zone boundary in the longest path, see figure 6.5(b). Given this stochastic nature of the simulation, each time $\mu(k = 0)$ is moved further away from the design target, 2000 experimental runs are made to ensure deterministic effects of the pseudo-random number generation are eliminated.

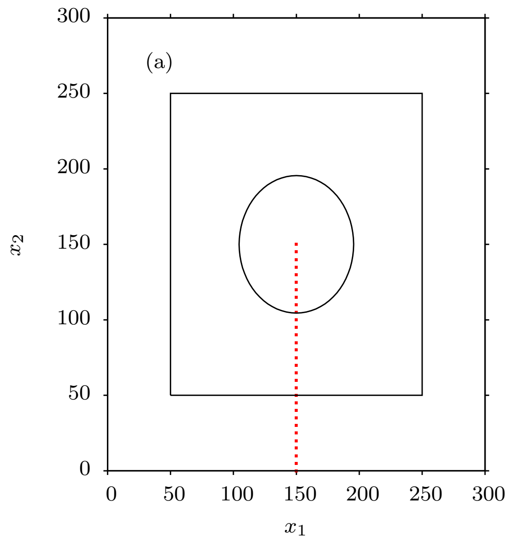


Figure 6.5: Starting positions of $\mu$ in the correlated simulated process: (a) taking the shortest path to the red zone; (b) taking the longest path to the red zone.

There are two results from these experiments: the probability of qualifying a process and the number of samples to make this decision. The probability of qualifying a process without any adjustments being made is shown in figure 6.6. The number of samples needed to make this decision is shown in figure 6.7.

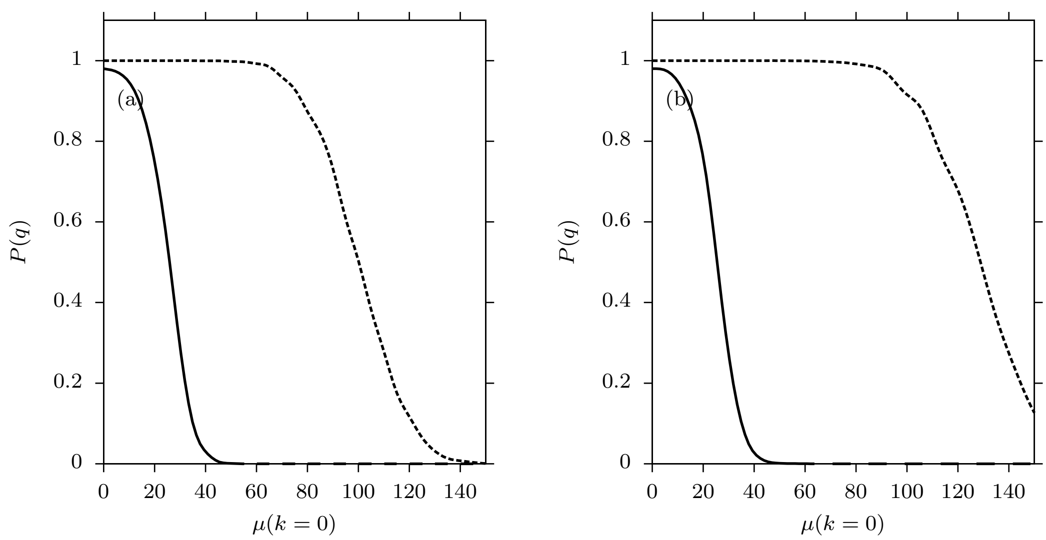


Figure 6.6: Operating curve of probability of qualifying ($P(q)$) without any adjustments against the position of $\mu(k=0)$ as a distance from process target for: (a) shortest path; (b) longest path.

In figure 6.6(a) the $P(q)$ is shown for the situation where $\mu(k=0)$ is moved away from the design target taking the shortest path to the red zone. In figure 6.6(b) the $P(q)$ is shown for the situation where $\mu(k=0)$ is moved away from the design target taking the longest path to the red zone. The mSUPA results are plotted with a solid line and the first-off results are plotted with a dotted line. Both these figures demonstrate that when the process is on-target, i.e. $\mu(k = 0) = 0$, mSUPA qualifies or validates approximately 98% of the processes, whereas first-off validates 100% of the processes. This is a critical result, as it demonstrates that mSUPA maintains the same confidence level as SUPA. Figure 6.6 also showed that when $\mu(k = 0)$ moved away from the design target on the longest path, a slightly higher percentage of processes are validated in both cases. Further, as $\mu(k = 0)$ moved away from the design target the operating curve for the SUPA rejected more process than the operating curve for the first-off method, highlighting that SUPA is more able to detect an off-target process. However, the difference in results between the shortest and longest path are similar when $\mu(k = 0)$ is close to 0 and, although different as $\mu(k = 0)$ moves away from 0, this difference is not large.

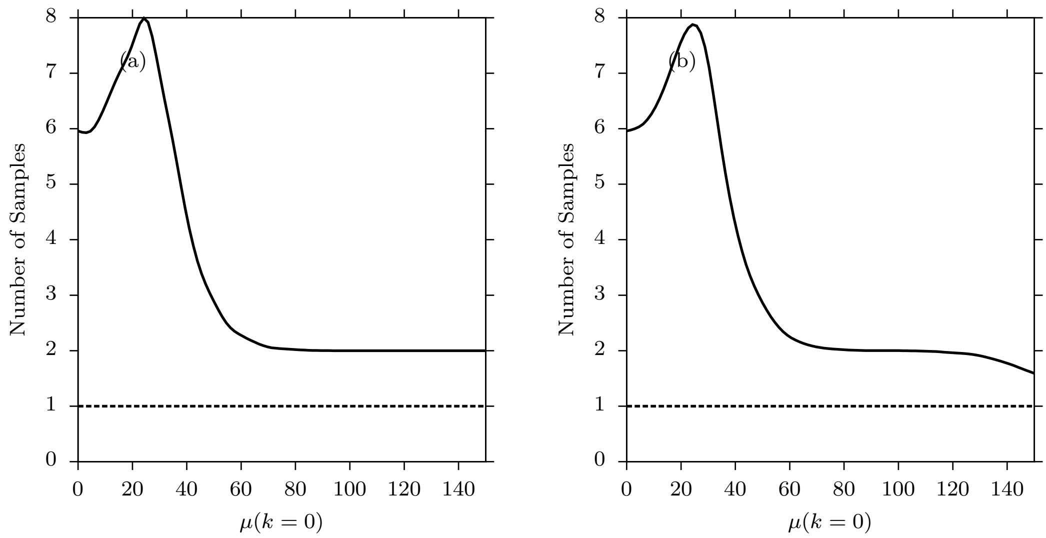


Figure 6.7: Operating curves of the number of samples to make a decision against $\mu(k = 0)$ for: (a) shortest path; (b) longest path.

In figure 6.7 The number of samples to make a valid/off-target decision is shown for the situation where $\mu(k = 0)$ is moved away from the design target, with (a) taking the shortest path to the red zone and (b) taking the longest path to the red zone. The mSUPA results are plotted with a solid line and the first-off results are plotted with a dotted line. This shows that mSUPA performs in a similar manner in both situations. It highlights that an on-target process, where $\mu(k = 0) = 0$, is validated within six samples. As $\mu(k = 0)$ moves away from the design target increasingly more samples are required to make a decision, until a peak of just under 8 samples at $\mu(k = 0) \approx 30$. Then there is a drop off in the number of units required down to two samples at $\mu(k = 0) \approx 60$. This behaviour reflects mSUPA requiring more units to make a decision as $\mu(k = 0)$ approaches the green/yellow zone boundary. Once this boundary has been crossed mSUPA only requires two samples to identify that the process is off-target. These fluctuations in the number of samples required are consistent with the results obtained for univariate SUPA. The first-off method, however, only ever requires a single sample by definition. Although, first-off is the more efficient approach, figure 6.6 clearly demonstrates that this compromises its performance.

In a final experiment, starting process mean positions, $\mu(k=0)$, were uniformly distributed across the design space. 600,000 simulations were run to produce a concentration diagram of $\mu(k=0)$ positions that were validated. These concentration diagrams are shown in figure 6.8.

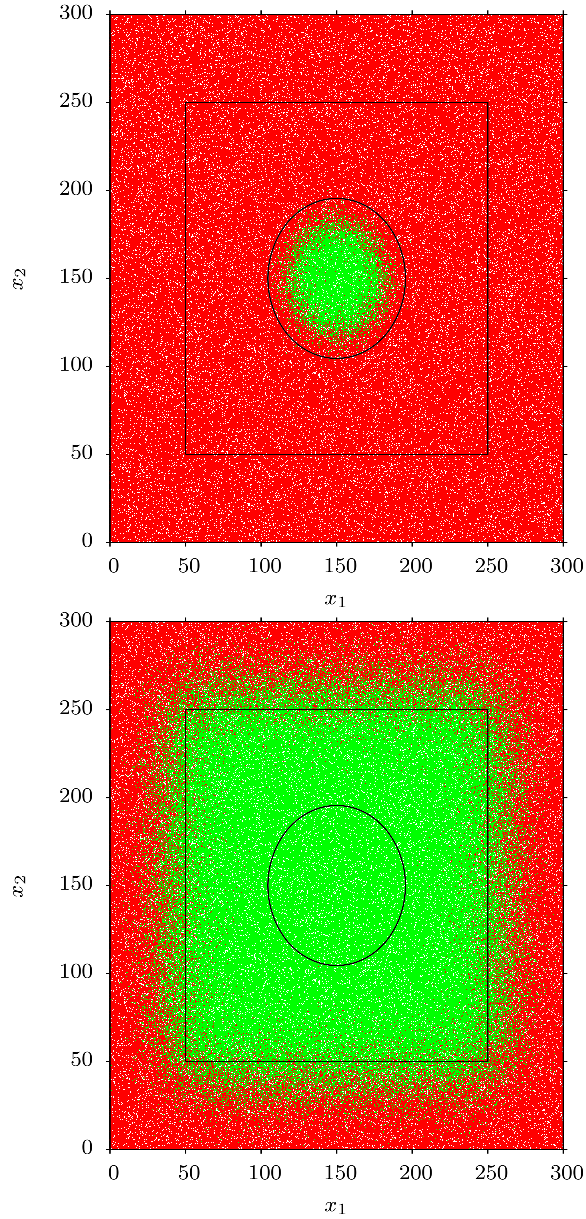


Figure 6.8: Concentration diagrams showing $\mu(k=0)$ positions with green representing a validated process and red representing an off-target process.

In figure 6.8 processes that were validated are plotted in green and processes that signal an adjustment was required are plotted in red. The first concentration diagram shows results using mSUPA. This clearly shows that as $\mu(k=0)$ approaches the green/yellow zone boundary, fewer processes are validated. As $\mu(k=0)$ approaches the yellow/red zone boundary, no processes are validated. This result demonstrates that by using mSUPA off-target processes are identified before $\mu(k=0)$ is outside the green zone, which is determined by the minimum capability required from the process. The second concentration diagram shows results using the first-off method. This clearly shows that as $\mu(k=0)$ approaches the yellow/red zone boundary, fewer processes are validated. However, there are a significant number of valid green points falling outside the tolerance of $x$. This result confirms the results shown in figure 6.6 and demonstrates that by using first-off method off-target processes are identified after $\mu(k=0)$ is outside the tolerances.

### 6.5.3 Two Correlated CtQs

In this series of experiments, the ability of mSUPA and the first-off method to detect an off-target process is tested when there is a correlation between individual CtQ. The process is also producing CtQ features with a capability $C_p = 2.0$. This means for this example that the processes covariance matrix, $\Sigma$, is:

$$\Sigma = \begin{bmatrix} 277.78 & 250 \\ 250 & 277.78 \end{bmatrix} \tag{6.8}$$

In the first experiment, the CtQs process means were set equal to the design target, i.e. $\mu = T$. The simulation was then allowed to run uninterrupted, producing 10,000 products. Then the resulting $x$ values were plotted to visualise the distribution of points created by $\Sigma$ value in equation 6.8. These results are displayed in figure 6.9, which shows a dense elliptical scattering of points within the green zone.

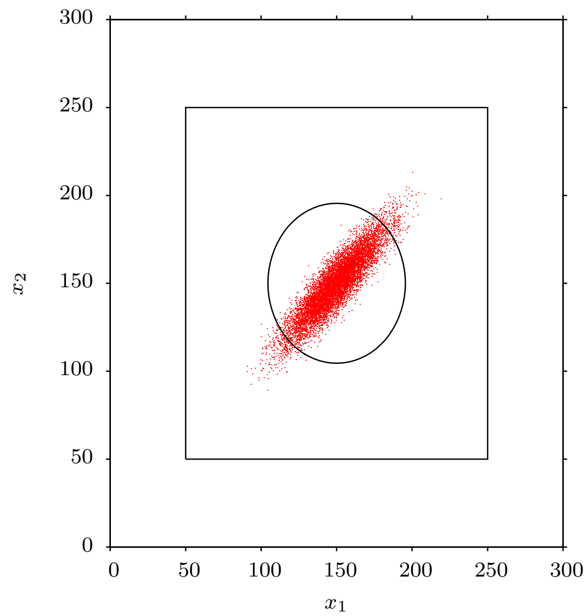


Figure 6.9: Distribution of $x$ values from the correlated simulated process.

This dense elliptical scattering of points is consistent with two features that are jointly distributed, i.e. there is a strong correlation between the two CtQ features. This confirms that the simulation is producing products with CtQ features as expected.

As in the previous case, a second experiment was then run with this simulation; whereby, the starting process mean ($\mu(k = 0)$) was moved progressively further away from the design target. Given the elliptical distribution of $x$ values in relation to the rectangular yellow/red zone boundary, $\mu(k = 0)$ is moved away from the design target in three directions. In the first run, the position of $\mu(k = 0)$ was moved away from the design target in a direction that crosses the red zone boundary in the shortest path, see figure 6.10(a). In the second run, the position of $\mu(k = 0)$ was moved away from the design target in a direction that crosses the red zone boundary in the shortest path from edge of the ellipse of points, see figure 6.10(b). In the third run, the position of $\mu(k = 0)$ was moved away from the design target in a direction that crosses the red zone boundary in the longest path, see figure 6.10(c). Given this stochastic nature of the simulation, each time $\mu(k = 0)$ is moved further away from the design target, 2000 experimental runs are made to ensure deterministic effects of the pseudo-random number generation is eliminated.

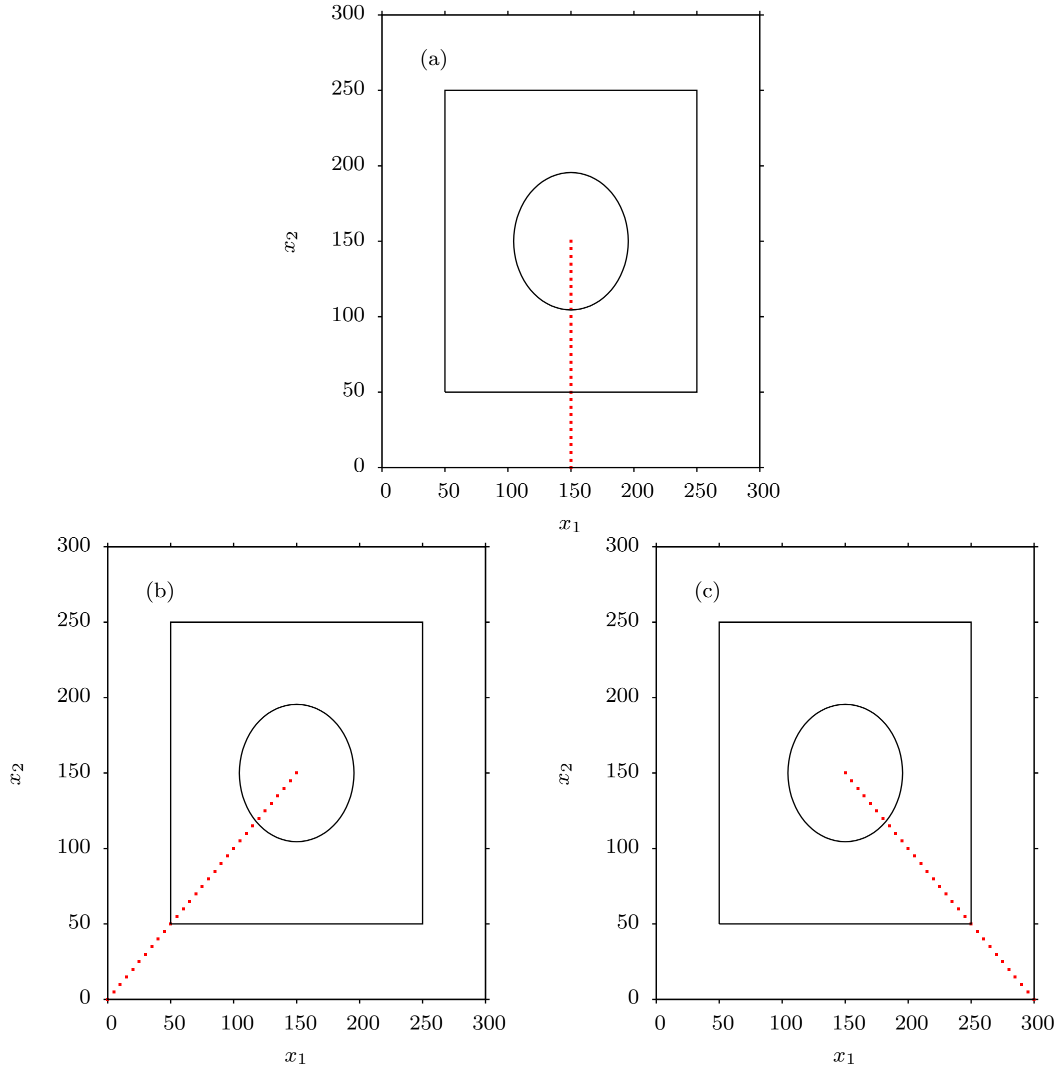


Figure 6.10: Starting positions of $\mu$ in the correlated simulated process: (a) taking the shortest path from the distribution centre to the red zone; (b) taking the shortest path from the tip of the distribution ellipse to the red zone; (c) taking the longest path from the distribution centre to the red zone.

Following the same pattern of experiments as in the uncorrelated case, there are two results from these experiments: the probability of qualifying a process and the number of samples to make this decision. The probability of qualifying a process without any adjustments being made is shown in figure 6.11. The number of samples needed to make this decision is shown in figure 6.12.

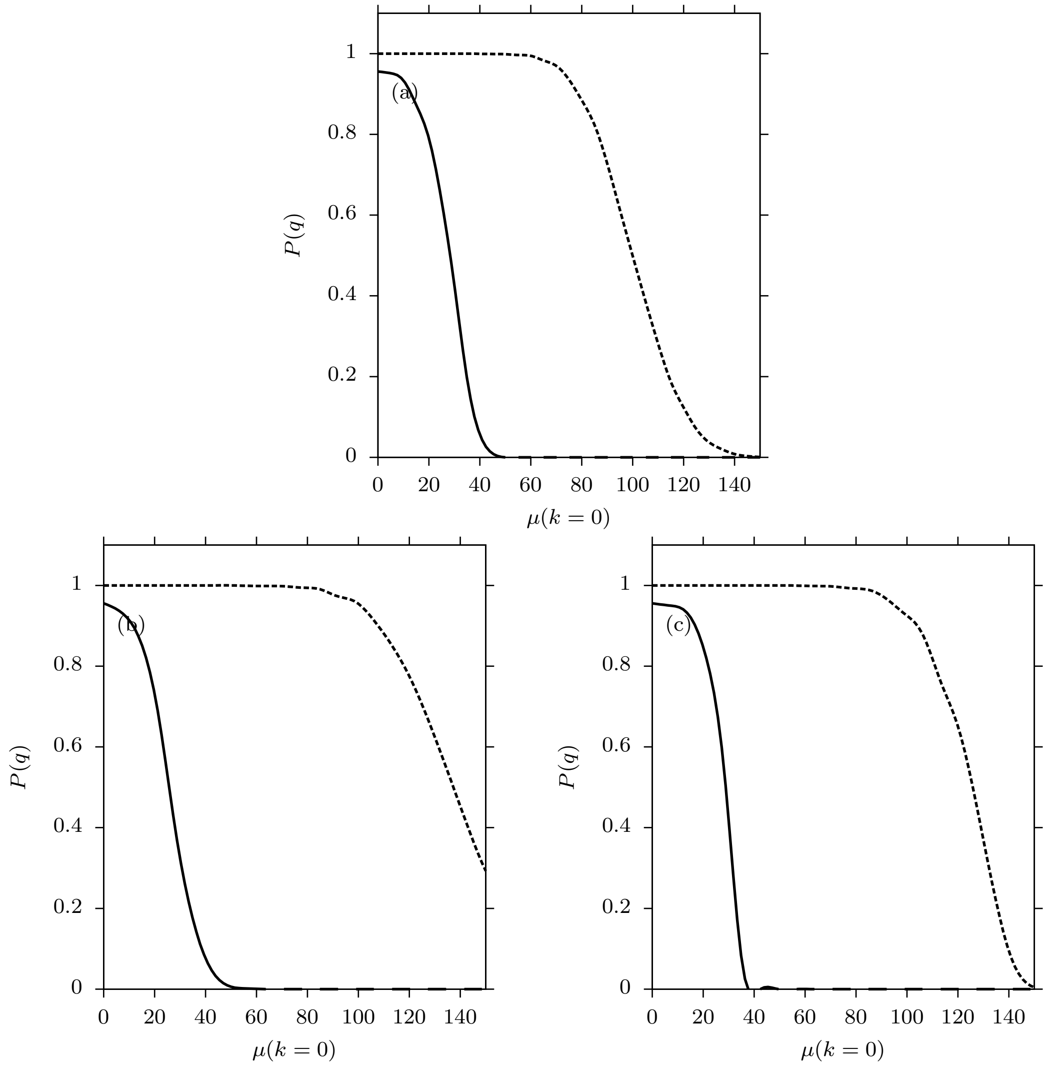


Figure 6.11: Operating curve of probability of qualifying ($P(q)$) without any adjustments against the position of $\mu(k=0)$ as a distance from process target for: (a) taking the shortest path from the distribution centre to the red zone; (b) taking the shortest path from the tip of the distribution ellipse to the red zone; (c) taking the longest path from the distribution centre to the red zone.

In figure 6.11, the mSUPA results are plotted with a solid line and the first-off results are plotted with a dotted line. The results in figure 6.11 demonstrate that when the process is on-target, i.e. $\mu(k = 0) = 0$, mSUPA validates approximately 98% of processes and first-off validates 100% of processes. This is consistent with the uncorrelated case and is a critical result as it, again, demonstrates that mSUPA maintains the same confidence level as SUPA. For mSUPA, there was little difference in performance between the results shown in figure 6.11(a;b); however, figure 6.11(c) showed that when $\mu(k = 0)$ moved away from the design target on the longest path, a higher percentage of processes are validated. This is demonstrated by figure 6.11(a;b) validating close to 98% of processes until $\mu(k = 0) = 10$; whereas, in figure 6.11(c) close to 98% of processes are validated until $\mu(k = 0) = 20$. For first-off, figure 6.11(a;b;c) shows 100% of processes were validated until $\mu(k = 0) = 60$ in all cases. When $\mu(k=0)=60$, mSUPA validate 0% of processes in all cases. This highlights that the first-off method is less able to detect deviations in the process mean when compared to mSUPA.

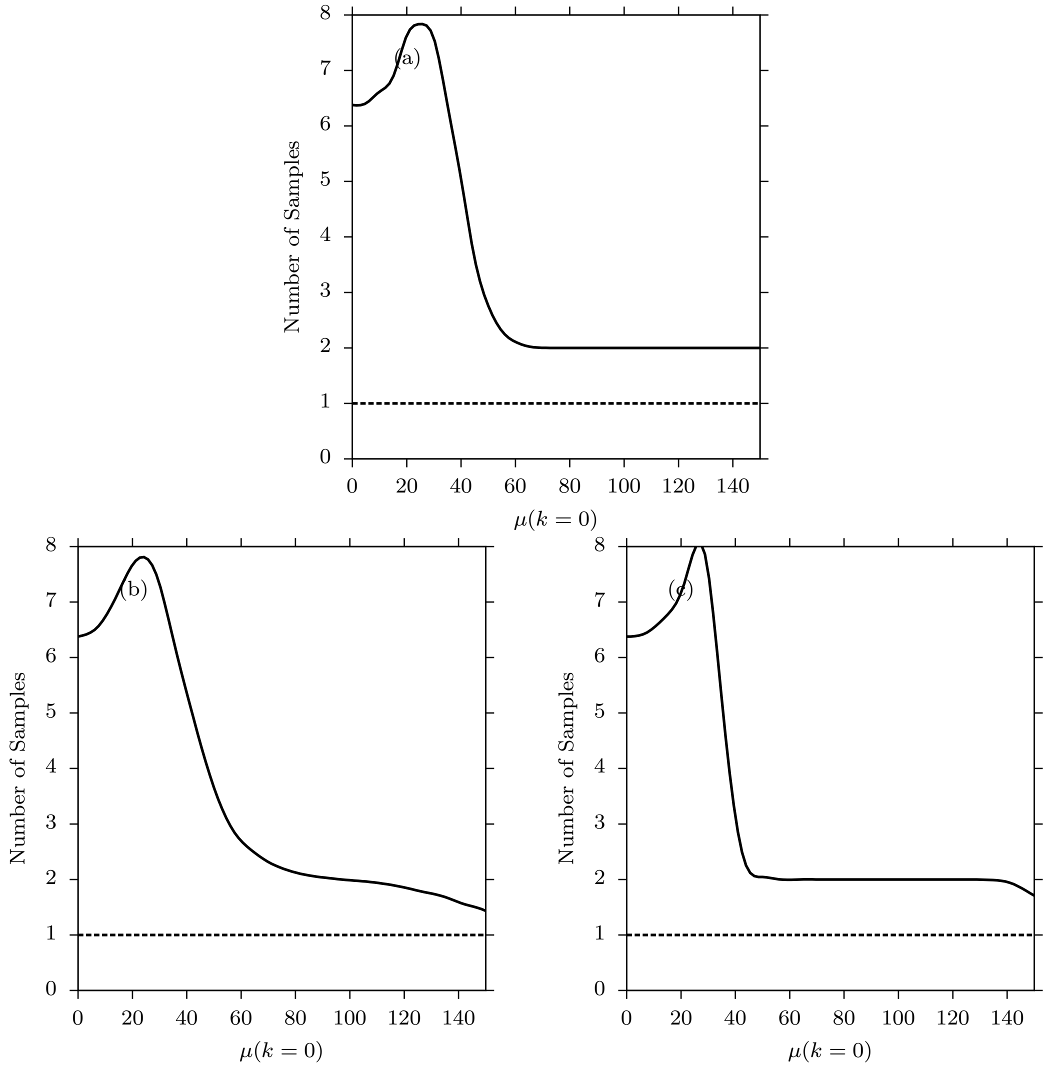


Figure 6.12: Operating curves of the number of samples to make a decision against $\mu(k=0)$ for: (a) taking the shortest path from the distribution centre to the red zone; (b) taking the shortest path from the tip of the distribution ellipse to the red zone; (c) taking the longest path from the distribution centre to the red zone.

In figure 6.12 the number of samples to make a valid/off-target decision is shown for the situation where $\mu(k=0)$ is moved away from the design target. There is very little difference between the results for mSUPA in figure 6.12(a;b); which, shows when $\mu(k=0)=0$ typically 6.4 samples are needed to make a decision. As $\mu(k=0)$ moves away from the design target increasingly more samples are required to make a decision, until a peak of just under 8 samples at $\mu(k=0)\approx 30$. Then there is a drop off in the number of samples required down to two samples at $\mu(k=0)\approx 50$. However, the results in figure 6.12(c) show when $\mu(k=0)=0$ typically 6.4 samples are needed to make a decision. As $\mu(k=0)$ moves away from the design target increasingly more samples are required to make a decision, until a sharper peak of just over 8 samples at $\mu(k = 0) \approx 30$. Then there is a drop off in the number of samples required down to two samples at $\mu(k = 0) \approx 50$. The drop off in the number of samples at $\mu(k = 0) \approx 50$ reflects the behaviour of mSUPA requiring more units to make a decision as $\mu(k = 0)$ approaches the green/yellow zone boundary. Once this boundary has been crossed mSUPA only requires two samples to identify that the process is off-target. These fluctuations in the number of samples required are consistent with the results obtained for univariate SUPA. The first-off method, however, only ever requires a single sample by definition. Although, first-off is the more efficient approach, figure 6.12 clearly demonstrates that this compromises its performance.

In a final experiment, starting process mean positions, $\mu(k = 0)$, were uniformly distributed across the design space. 600,000 simulations were run to produce a concentration diagram of $\mu(k = 0)$ positions that were validated. This concentration diagram is shown in figure 6.13.

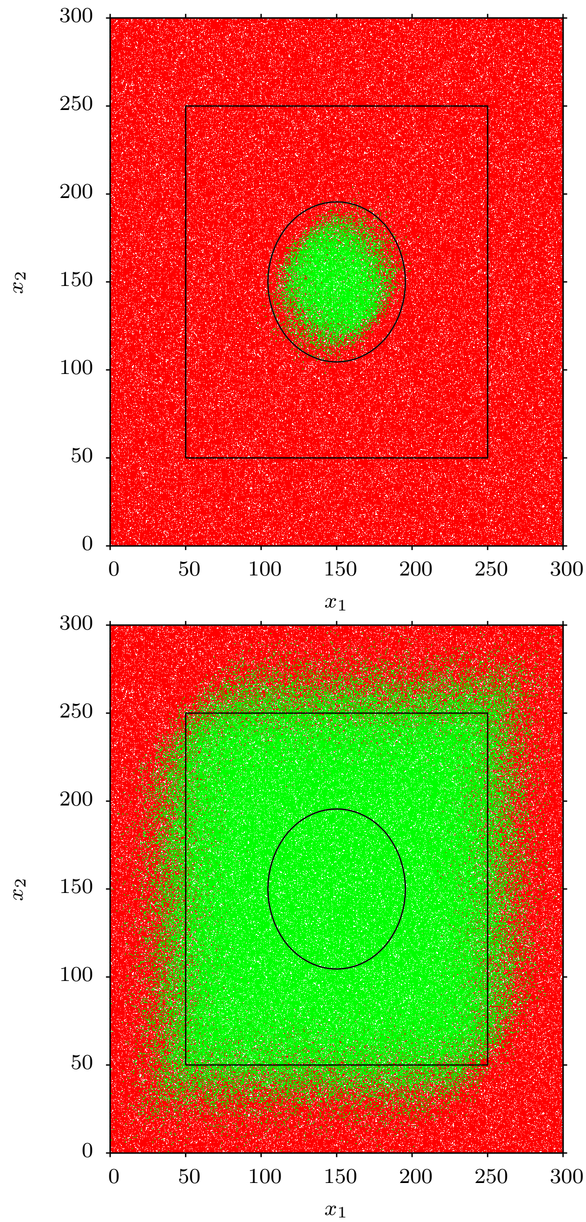


Figure 6.13: Concentration diagrams showing $\mu(k = 0)$ positions with green representing a validated correlated process and red representing an off-target process.

In figure 6.13 processes that were validated are plotted in green and processes that signal an adjustment was required are plotted in red. The first concentration diagram for mSUPA clearly shows that as $\mu(k = 0)$ approaches the green/yellow zone boundary, fewer processes are validated. As $\mu(k = 0)$ approaches the yellow/red zone boundary, no processes are validated. It is also shown that the green points form an ellipse shape; however, this ellipse is contained within the green zone. This suggests that by using mSUPA off-target processes are identified before $\mu(k = 0)$ is outside the green zone irrespective of the underlying process distribution. The second concentration diagram shows results using the first-off method. This clearly shows that as $\mu(k = 0)$ approaches the yellow/red zone boundary, fewer processes are validated. However, there are a significant number of valid green points falling outside the tolerance of $x$ with more green points falling outside the tolerance when $x_1$ and $x_2$ are both low or high. This reflects the shape of the underlying distribution. This result confirms the results shown in figure 6.11 and demonstrates that by using first-off method off-target processes are identified after $\mu(k = 0)$ is outside the tolerances.

## 6.6 Summary

In this chapter, a new nonparametric process control tool was introduced known as Multivariate Set-Up Process Algorithm (mSUPA). mSUPA was designed to monitor multivariate set-up dominant processes; in order, to identify when the process mean is not on the global design target.

The need for such a tool was highlighted with a case study example in section 6.2. This case study used the example of a multi-axis CNC machining centre to identify challenges faced by low-volume production systems. It was identified that classical SPC tools were not sufficient to monitor parts with multiple CtQ features, that were produced in low-volumes. The primary issue identified with classical SPC tools was that it was not possible to obtain enough data from a production run to set-up statistical control limits. There were further potential issues identified associated with operators responsible for processes interpreting statistically derived control chart.

The company involved in the case study had recently employed Pre-Control charts to monitor their set-up dominant processes. There were clear benefits to using this approach: it did not require a start-up period to set up control limits; linking control limits to tolerance limits provided operators with an intuitive control tool. However, implementation of Pre-Control was not without its own issues. When applied to a multivariate process where that the parts being produced have multiple CtQ features, this requires the management of multiple Pre-Control charts[1]. This can quickly become unmanageable.

Further to this, the control rules that give Pre-Control statistical confidence in the control observations derived by them, could not be employed. Therefore, the company employed a 100% inspection regime. Firstly, Pre-Control ensures that a process is maintaining a minimum process capability $C_{pk} \geqslant 1.33$, for certain CtQs this may not be of a high enough standard. Secondly, due to an imbalance between the number of control parameters and CtQ features, there are situations where to keep one CtQ in a green zone another CtQ has to run in a yellow zone. The operators would therefore benefit from a global approach to process control rather than a feature by feature approach.

In order to overcome this situation, the mSUPA method was proposed in section 6.3. This extends the univariate SUPA method into the multivariate case. By doing this, an easy to digestive traffic-light system is maintained that is linked to tolerance boundaries. Unlike using Pre-Control, the mSUPA charts can be adjusted to suit a range of required process

[1] 26 CtQs in the case study example, results in 26 Pre-Control charts to manage.

capabilities. Although, the presentation of results through a simple global traffic-light to inform the operator that the process is on- or off-target is easy to digestive, it requires more complex calculations to decide which zone a part falls into. Therefore, it is recommended that this approach is implemented through an online software interface to perform the required calculations. An issue not addressed in this chapter, is how to optimise a process once it is identified as being off-target? This is an issue which is particularly complex in multivariate processes, where there are significantly less control parameters to adjust and there are CtQ features that they affect. This problem is addressed in the following chapter.

# Chapter 7

# Adjustment Simulator



## 7.1 Introduction

In chapter 5 a new method for control of a univariate set-up dominant process was introduced known as SUPA. This was extended, in chapter 6, to provide a method for control of a multivariate set-up dominant process known as mSUPA. These two process control methods provide operators of low-volume manufacturing processes with a means to identify when a process is off-target with statistical confidence. However, once a process has been identified as off-target, how it is then adjusted is left to the operator's experience or rule-of-thumb. This is not an issue in the univariate case, where there is typically an independent control parameter which directly affects the CtQ being monitored. But, the adjustment of process parameters in the multivariate case can pose a challenge, particularly when there are fewer process parameters to adjust there are CtQs to keep in control. This issue was highlighted in the case study in section 6.2, which described a single multi-axis machining centre manufacturing a part with 26 CtQ features. In that case, adjusting one machining centre offset affected two or more CtQs.

In the literature reviewed in section 3.4, a hybrid control system known as run-to-run control (RtR) was identified as a potential solution to the process adjustment issue. The generalised RtR approach integrates discrete process control tools, such as SPC, and continuous feedback controllers. Current RtR methods are not suitable for set-up dominant processes due to their use of classic SPC approaches. However, the philosophy of applying a discrete process control tool and continuous feedback controller does have potential in low-volume applications. Therefore, this chapter explores the potential of designing a new RtR method for set-up dominant processes which uses mSUPA as the discrete process control tool.

In section 7.2 the concept of direct feedback using feedback controllers is described and explored. This concept is expanded in section 7.3, where a simulated feedback approach is used. This simulated feedback approach is then tested in a simulation where there are less control parameters than there are CtQ design features in section 7.4. The simulated feedback approach is then developed further based on the experiments conducted in section 7.4 to provide an updated method in section 7.5.

## 7.2 Adjustment Feedback Loop

### 7.2.1 RtR Approach

In this section a new RtR method is introduced that utilises mSUPA, described in chapter 6, as the discrete process control tool in conjunction with feedback controllers. A basic flowchart describing this methodology is shown in figure 7.1.

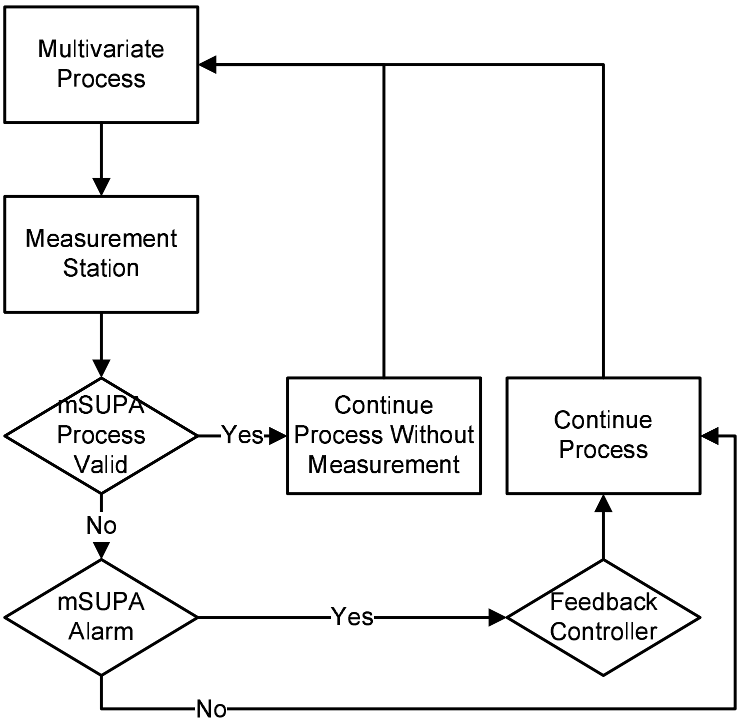


Figure 7.1: Flow chart of mSUPA based RtR method.

Figure 7.1 shows a multivariate process producing parts. These parts CtQ features are then checked at a measurement station. The data generated at the measurement station is then used to validate the process with the mSUPA method. If a process is validated, i.e. there has been five consecutive green parts, the process is allowed to run without the need of further checks on parts produced. If a process is not validated, mSUPA decides whether an alarm needs to be generated or not. If an alarm is not generated, the process is allowed to continue but consecutive parts continue to be checked at the measurement station. If an alarm is generated, i.e. there has been one red part or two consecutive yellow parts, feedback controllers are applied to optimise process parameters.

In order to model the relationship between the changes in control parameters and the resulting effect on the process output, data needs to be recorded regarding the time and size of current changes to control parameters settings. The relationship between the current process mean when part $k$ has been measured, $\mu(k)$, and the next process mean when part $k+1$ has been measured, $\mu(k+1)$, after an adjustment, is described as a discrete time linear dynamic system of the form:

$$\mu(k+1) = \mu(k) + A \cdot u(k) \tag{7.1}$$

where $u(k)$ is the $m$-dimensional vector of adjustments performed after part $k$ has been measured, and $m$ depends on the adjustment capabilities of the process, i.e. the ability of an operator to make different corrections on the machine; $A$ is an $n \times m$ matrix modelling the causal relationship between adjustment actions taken and their effect on every CtQ design feature. If the mSUPA chart indicates that an adjustment to the control factors is required, the size of adjustment $u(k)$ can be calculated by using a feedback control approach.

Depending on the properties and the setting of the machining centre, the number of independent adjustments may vary. For example, it may be possible for the operator to make corrections for each design feature individually, in which case the dimension of the control vector $u = [u_1, u_2, u_3]^T$ and $m = n = 3$. In this case, equation (7.1) can be expanded as follows:

$$\mu_1(k+1) = \mu_1(k) + a_{11}u_1(k) + a_{12}u_2(k) + a_{13}u_3(k) \tag{7.2}$$

$$\mu_2(k+1) = \mu_2(k) + a_{21}u_1(k) + a_{22}u_2(k) + a_{23}u_3(k) \tag{7.3}$$

$$\mu_3(k+1) = \mu_3(k) + a_{31}u_1(k) + a_{32}u_2(k) + a_{33}u_3(k) \tag{7.4}$$

where $a_{ij}$ are elements of the $A$ matrix. This simple set of linear equations could be solved to find the optimum control vector adjustment by setting $\mu_1(k+1) = T_1$, $\mu_2(k+1) = T_2$ and $\mu_3(k+1) = T_3$, where $T_1$, $T_2$ and $T_3$ are the targets for CtQ design features 1, 2 and 3, respectively. However, this situation where there are as many control parameters as there are CtQ features was simulated to demonstrate the potential of the proposed RtR method. It is shown in section 7.4 the effect of RtR under the conditions were there are fewer control parameters than CtQ features. In order to complete the RtR method, a proportional feedback controller is used for the case of $n = m = 3$. Assuming that $u$ is the adjustment made and $\bar{x}$ is the estimate of $\mu$, this leads to the following proportional feedback control calculations:

$$
\begin{aligned}
u_1(k) &= G_1[\bar{x}_1(k) - T_1] \\
u_2(k) &= G_2[\bar{x}_2(k) - T_2] \\
u_3(k) &= G_3[\bar{x}_3(k) - T_3]
\end{aligned}
\tag{7.5}
$$

where, $G_1$, $G_2$ and $G_3$ are gain factors. $G_1$, $G_2$ and $G_3$ are set to improve the dynamic response when an adjustment is required, i.e. they can be fine tuned. A change in the control factor $u_1(k)$, has an effect on mean of the CtQ design features $\mu_1$, $\mu_2$ and $\mu_3$ through the elements of matrix $A$, as modelled in equations (7.2)-(7.4). These feedback control actions are only made if consecutive units fall in the yellow or red zones of the mSUPA chart as per control rules of table 5.2. In the remainder of this section, the values of $G$ are adjusted to demonstrate their affect on proportional feedback control.

### 7.2.2 Simulation Example

In the following figures 7.2-7.6 the effect of this dynamic feedback approach is demonstrated with results from a simulation. In the simulation, the vector $x$ represents the three CtQ design features, $x_1$, $x_2$ and $x_3$; which, all have the same design target and tolerances of $L = 50$, $T = 150$ and $U = 250$. It is also required that each CtQ feature maintains a minimum $C_p \geqslant 2.0$. The next step is to specify the mSUPA-chart based on these parameters. Therefore, the mSUPA covariance matrix, $S$, is defined as:

$$
S = \begin{bmatrix} 277.778 & 0 & 0 \\ 0 & 277.78 & 0 \\ 0 & 0 & 277.78 \end{bmatrix} \tag{7.6}
$$

Using this value for $S$, a decision can be made as to whether an $x$ value falls in a green zone by solving the inequality in equation (6.3). However, to solve this inequality a value for $H^2$ is needed. Given that this value has the property following a $\chi^2_{n,\alpha}$ distribution; where, the degrees of freedom $n = 3$ and the probability that an on-target process produces an $x$ value outside the green zone $\alpha = 0.06$. Hence, $H^2 = \chi^2_{3,0.06} = 7.412$.

The process in the simulation maintains the multivariate normal distribution of:

$$
x \sim N(\mu, \Sigma) \tag{7.7}
$$

The initial position of the process mean, $\mu(k = 0)$, is started off-target. This is to demonstrate the effect of the proportional feedback controllers at steering the process back on-target. In the simulation results presented in figures 7.2-7.6, the process is started at:

$$
\mu(k = 0) = \begin{bmatrix} 215 \\ 60 \\ 150 \end{bmatrix} \tag{7.8}
$$

The covariance matrix, $\Sigma$, remains constant throughout the simulation runs and is set to:

$$\Sigma = \begin{bmatrix} 250 & 200 & 170 \\ 200 & 250 & 150 \\ 170 & 150 & 250 \end{bmatrix} \tag{7.9}$$

This value of $\Sigma$ demonstrates correlations between the CtQ features in the $x$ vector. The control matrix, $A$, which links changes to control parameters, $u$, to a change in the process mean, $\mu$, as in equation (7.1), is also consistent throughout the simulation runs. In the simulation used in this section, $A$ is set to:

$$A = \begin{bmatrix} 78 & 45 & 0 \\ -24 & 93 & 15 \\ 25 & -16 & 86 \end{bmatrix} \tag{7.10}$$

In practice, this $A$ matrix is a function of the process that cannot be changed and the RtR method has to deal with it. If the linear equations formed by equation (7.2)-(7.4) where solved, $u(k) = [1.1857, 0.5885, 0.4542]^T$. However, this section simulates feedback controllers as described.

The final parameters that need to be specified are the gain values, $G$, of the proportional feedback controllers. In each simulation run in this section, the $G$ values are changed to demonstrate the effect of the gain values on the results achieved by the proportional feedback controllers. The results of the first simulation run were generated using the following $G$ values: $G_1 = 0.000125$, $G_2 = 0.000125$ and $G_3 = 0.000125$. By using these values for $G$, simulation results were generated and are presented in figure 7.2.

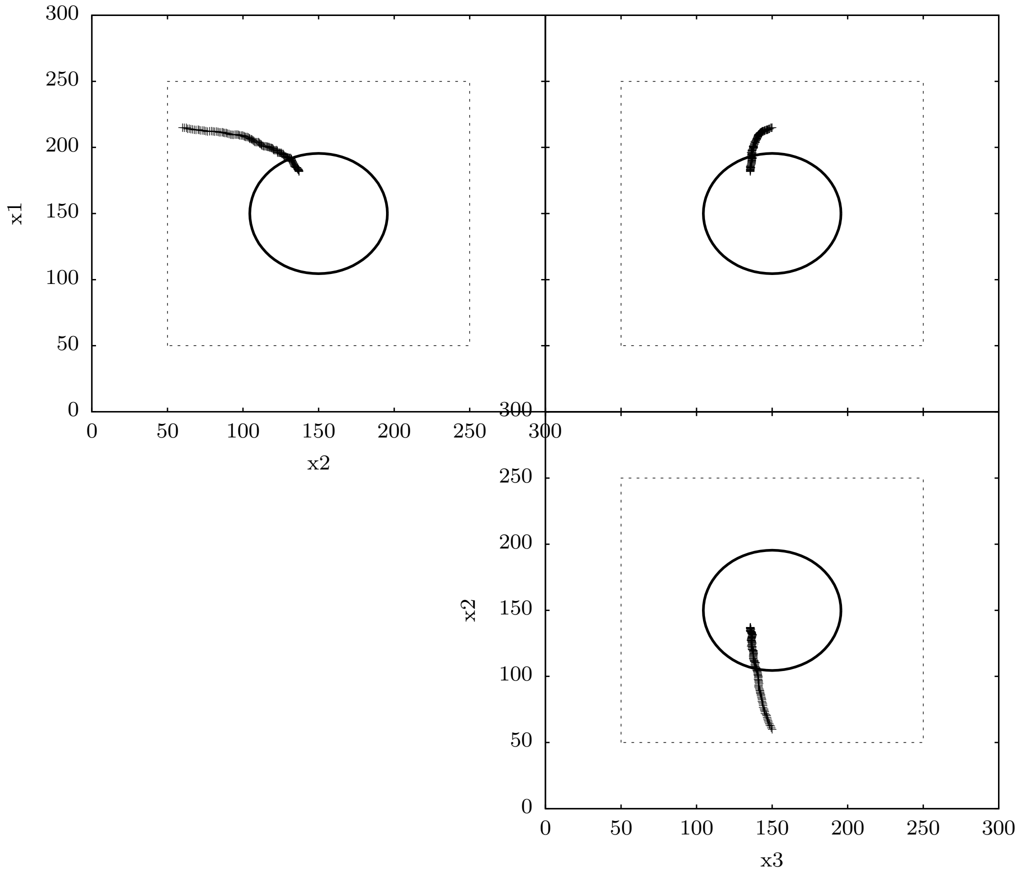


Figure 7.2: Path of control adjustments for a proportional feedback controller using $G = 0.000125$.

Figure 7.2 shows the path of the control adjustments with a line and the effect on the process mean, $\mu$, with crosses. This figure demonstrates that the proportional feedback controllers are capable of of driving a process that is off-target towards the design target. However, this figure also demonstrates that for the $G$ values used, there was a total of 122 control adjustment before the process was validated and this required a total of 323 products to be collected. Clearly, this is not a desirable situation, particularly in the context of low-volume manufacturing were production runs a commonly less than 50 products and can be as few as 5 to 10 products. Ideally, only one control adjustment would be made to get the process to validate.

Increasing the gains values should increase the step size of control adjustments, thus, reducing the number of control adjustments needed to drive the process to a valid position. Therefore to improve the situation presented in figure 7.2, the $G$ values were increased to:

$G_1 = 0.0015625$, $G_2 = 0.0015625$ and $G_3 = 0.0015625$. The simulation was then run again with these new $G$ values, the results are presented in figure 7.3.

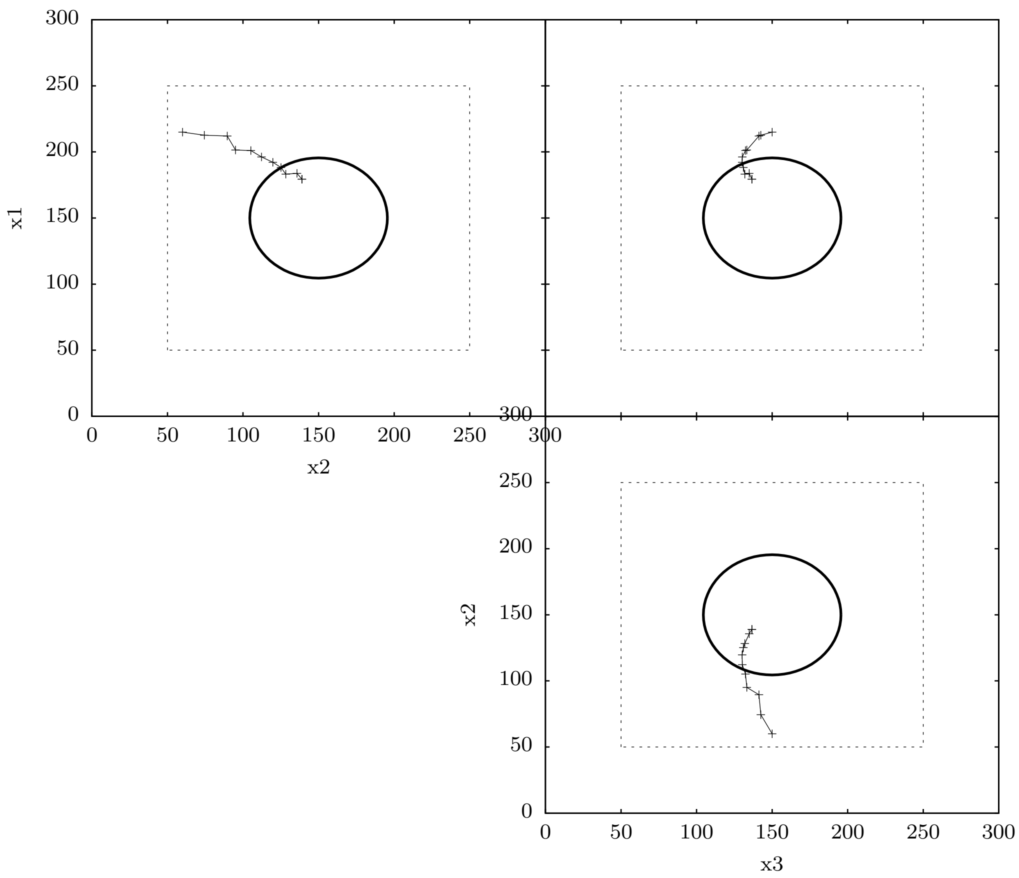


Figure 7.3: Path of control adjustments for a proportional feedback controller using $G = 0.0015625$.

Figure 7.3 shows the path of the control adjustments and the effect on $\mu$. As in figure 7.2, the results in figure 7.3 show that the control adjustments made by the proportional feedback controllers are driving $\mu$ towards the design target. In this simulation case there was a total of 11 control adjustments before the process was validated and this required a total of 52 products to be collected. The result of using larger $G$ values has significantly reduced the number of the control adjustments applied. However, this is still a large number of control adjustments in the context low-volume manufacture.

The $G$ values were increased, in order to reduce the number of the control adjustments needed. The next simulation used $G$ values of: $G_1 = 0.00625$, $G_2 = 0.00625$ and $G_3 = 0.00625$. The results produced by using these $G$ values are plotted in figure 7.4.

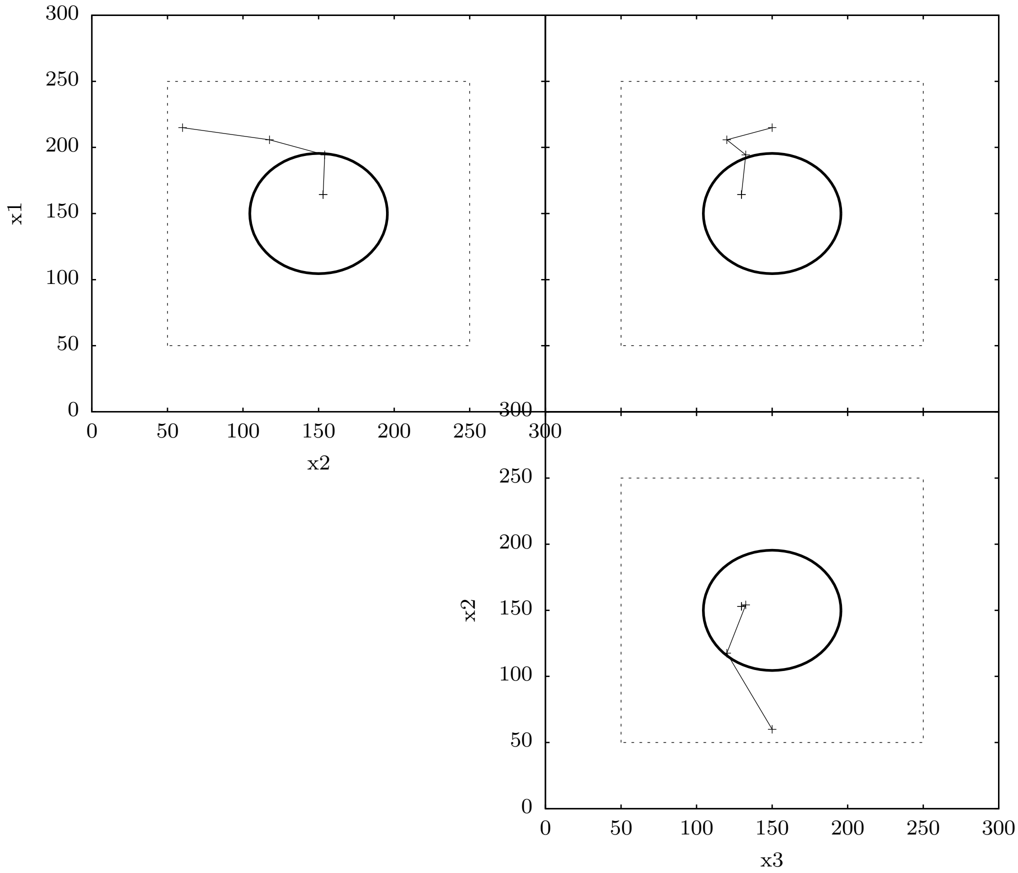


Figure 7.4: Path of control adjustments for a proportional feedback controller using $G = 0.00625$.

In figure 7.4 it is shown that by increasing $G$ from 0.0015625 to 0.00625, the number of control adjustments made by the proportional feedback controllers was reduced. In this simulation case there was a total of 4 control adjustments before the process was validated and this required a total of 15 products to be collected. Again, increasing the $G$ values are significantly reduce the number of control adjustments applied and also the number of products collected.

In the next simulation run the $G$ values were increased, in order to reduce the number of control adjustments further. The following $G$ values were used: $G_1 = 0.0125$, $G_2 = 0.0125$ and $G_3 = 0.0125$. The results from this simulation are presented in figure 7.5.

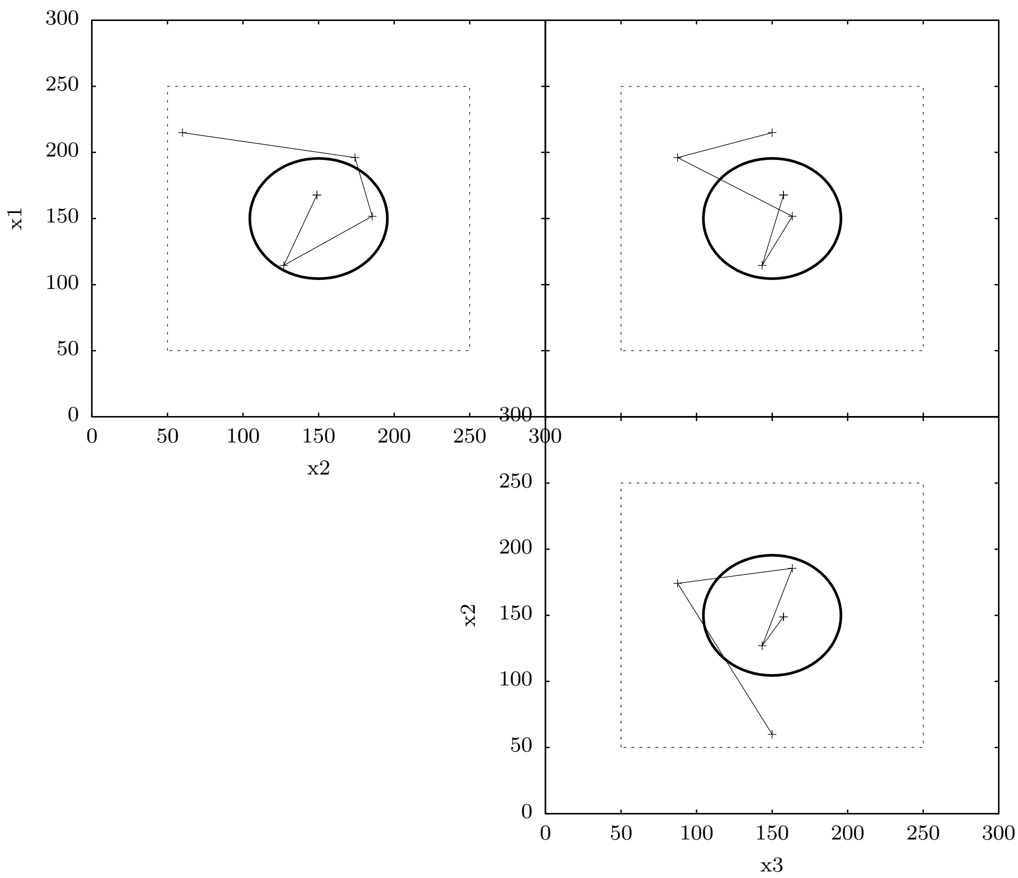


Figure 7.5: Path of control adjustments for a proportional feedback controller using $G = 0.0125$.

Figure 7.5 shows the number of control adjustments made by the proportional feedback controllers has remained the same as in the previous simulation. Hence, in this simulation case there was a total of 4 control adjustments before the process was validated and this requires a total of 16 products to be collected. These results indicate a very marginal difference in performance between a $G = 0.0015625$ and $G = 0.0125$. However, if figures 7.4 and 7.5 are compared, the path of $\mu$ directed by the control adjustment is very different. Figure 7.4 takes a direct route towards the design target; whereas, figure 7.5 takes a less stable route around the design target.

The $G$ values are increased again in the next simulation run to: $G_1 = 0.025$, $G_2 = 0.025$ and $G_3 = 0.025$. These results are presented in figure 7.6.

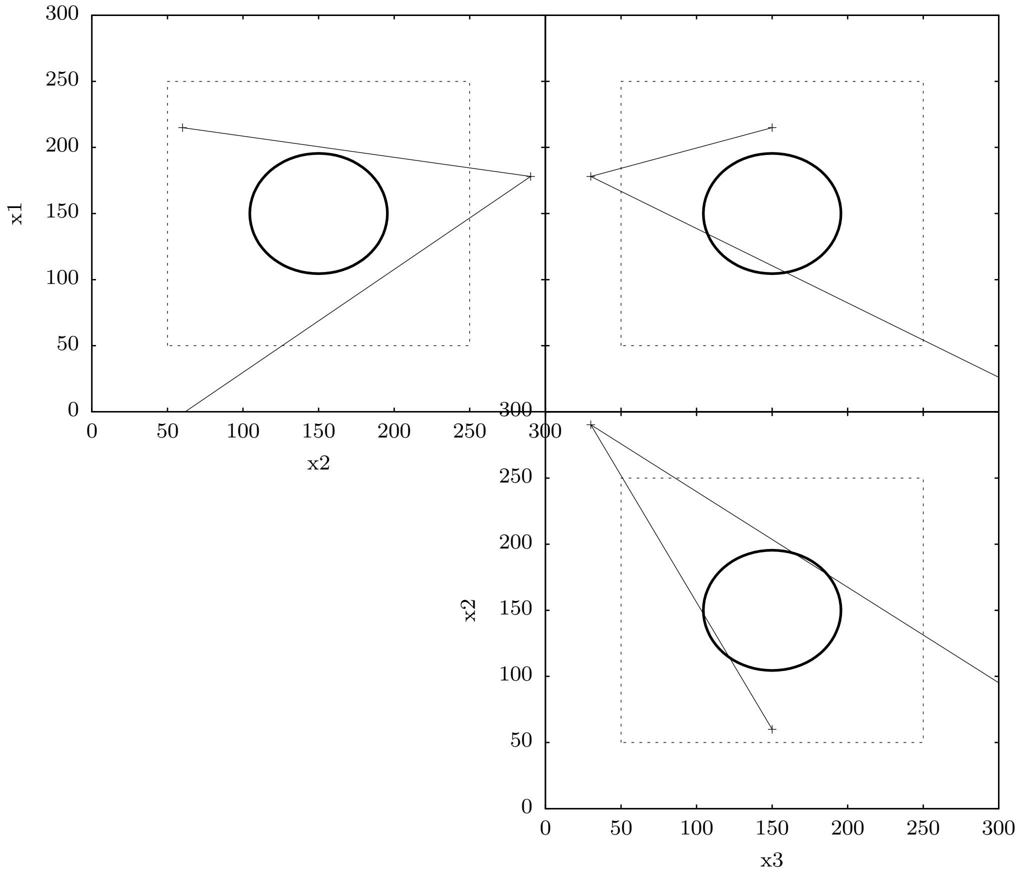


Figure 7.6: Path of control adjustments for a proportional feedback controller using $G = 0.025$.

In figure 7.6 it is shown that the number of control adjustments made by the proportional feedback controller has actually increased when compared to the previous simulation. In this case the simulation did not stop, it kept making control adjustments, collecting more and more products. The path of $\mu$ directed by the control adjustments spirals around the design target; except rather than converging on the design target, $\mu$ moves further away. This simulation demonstrates that if the $G$ values used are too large the path of $\mu$ becomes extremely unstable and rather than improving the process makes the situation worse.

This section has shown that the use of proportional feedback controllers in conjunction with mSUPA has the potential to drive the process mean, $\mu$, towards its design target through iterative control adjustments. The key issue uncovered is the effect of the gain values, $G$, on the efficiency and effectiveness of a proportional feedback controllers. If the $G$ values are small, the proportional feedback controller provides a steady path converging on the design target; however, it requires a large number of control adjustments and products to be collected. If the $G$ values are large, the proportional feedback controller provides an unsteady path and if $G$ values are very large, the proportional feedback controller does not converge on the design target. In the next section, this problem is addressed using a novel simulation-based approach.

## 7.3 Simulated Feedback Approach

### 7.3.1 Direct Feedback

In the previous section a new RtR approach that combined mSUPA and proportional feedback controllers was introduced. This RtR approach is outlined in figure 7.1. The key issue identified with the method was the balance between using a $G$ value that drives a process towards its design target in a direct and stable trajectory and a $G$ value that is less stable but more efficient. Another issue is that although the proportional feedback controllers are capable of driving a process towards its design target, it is prevented from doing so by the mSUPA method. If the previous example of a process with $\mu(k=0) = [216, 60, 150]^T$ is simulated with a feedback controller $G = [0.001, 0.001, 0.001]^T$ and the simulation is repeated 1000 times, to minimise deterministic effects; the final validated $\mu$ positions are recorded in figure 7.7.

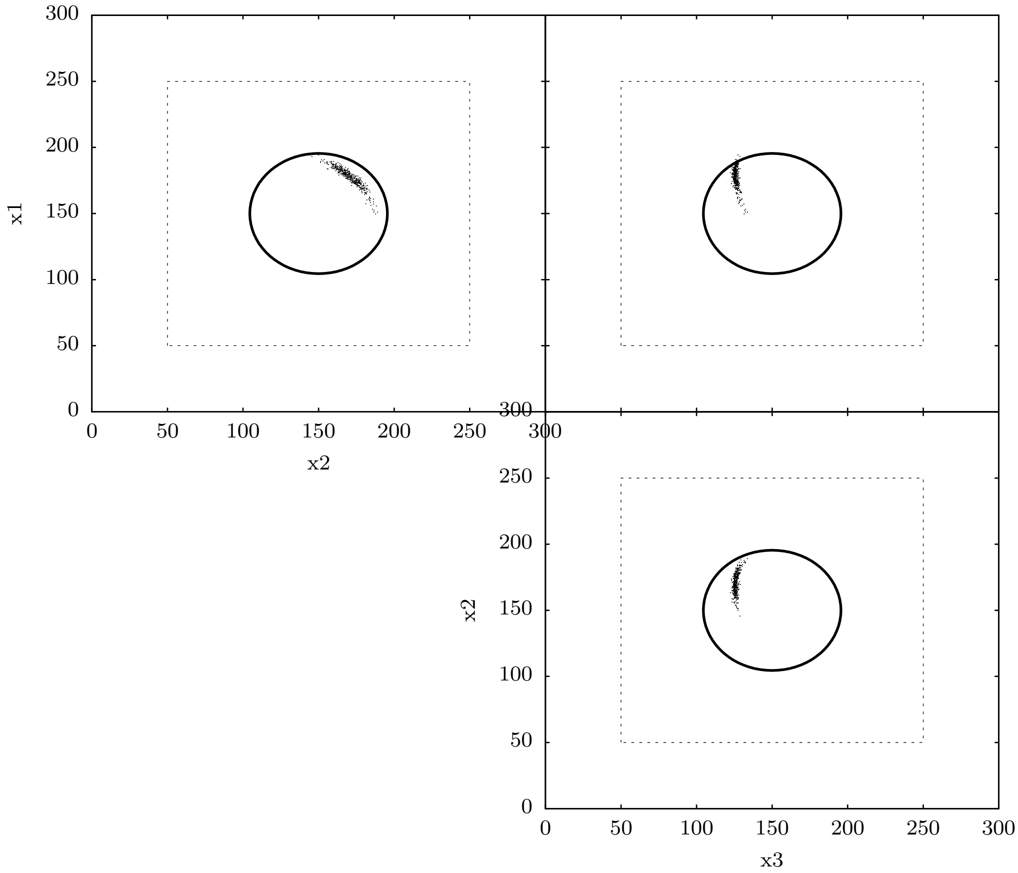


Figure 7.7: Final $\mu$ positions from RtR with direct proportional feedback control.

Figure 7.7 shows that the proportional feedback controllers are consistently driving the process into the mSUPA green zone. However, the process final validated $\mu$ positions are not on-target. The small size of the $G$ values used has also resulted in an average of 174.635 control adjustments being made to move $\mu$ from its initial position to its final dated position. This also required a mean of 465.911 products being collected.

These results support those made in the previous section that a low $G$ value, produces a large number of control adjustments and, therefore, a large number of products to be collected. However, these results also highlight that mSUPA is effectively cutting short the proportional feedback controllers ability to drive the process towards its design target. Therefore, a modified RtR approach is outlined next, which incorporates simulated proportional feedback controllers in an effort to maximise performance and minimise the number of control adjustments and products collected.

### 7.3.2 Simulated Feedback

The remainder of this section outlines an online simulation approach to applying proportional feedback controllers within an RtR method. The goal of the online simulation is to capture the benefit of using a proportional feedback controller with a small $G$ value, i.e. a stable path towards the design target, whilst minimising the number of control adjustments required to achieve this. An outline of this mSUPA based simulated adjustment RtR method is in figure 7.8.

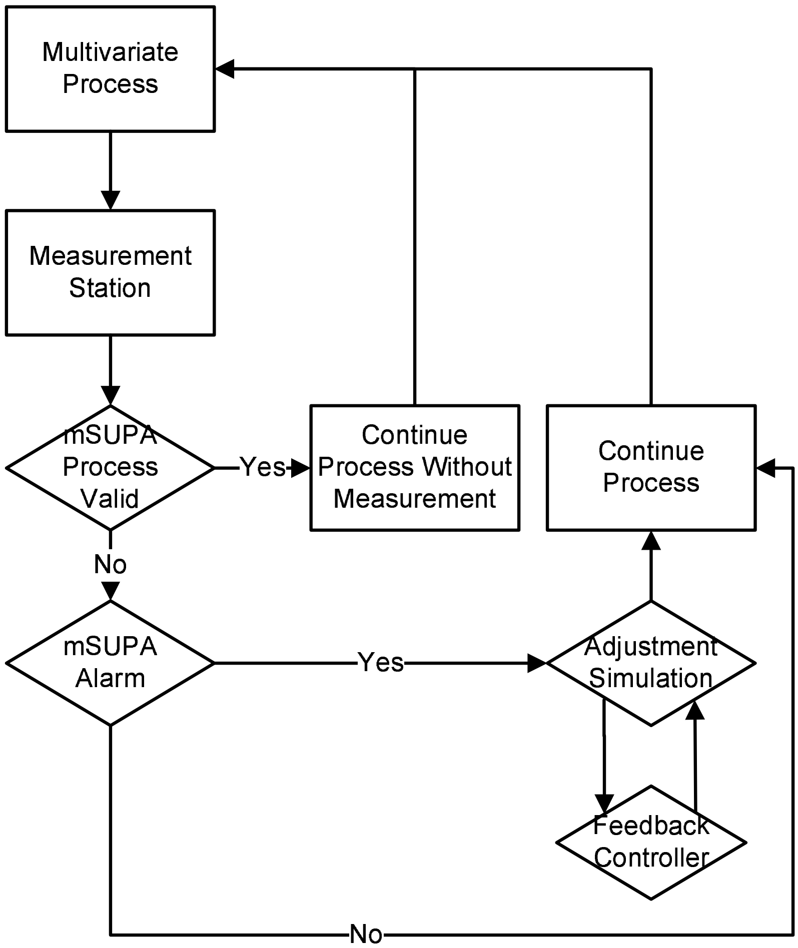


Figure 7.8: Flow chart of mSUPA based simulated adjustment RtR method.

Figure 7.8 shows: the process producing a part; this part's CtQs are then measured; these measurements are then plotted on an mSUPA chart. Once consecutive parts have been measured and plotted on the mSUPA chart, the process is either validated after five consecutive green parts or an adjustment is signalled after two consecutive yellow parts or a single red part. If the process is validated, production continues without further measurements being taken. If an adjustment is required, the size of the adjustment to the control parameters, $u$, is determined through a real-time online simulation using proportional feedback controllers. The calculation of these adjustments to the control parameters through simulation is outlined in figure 7.9.

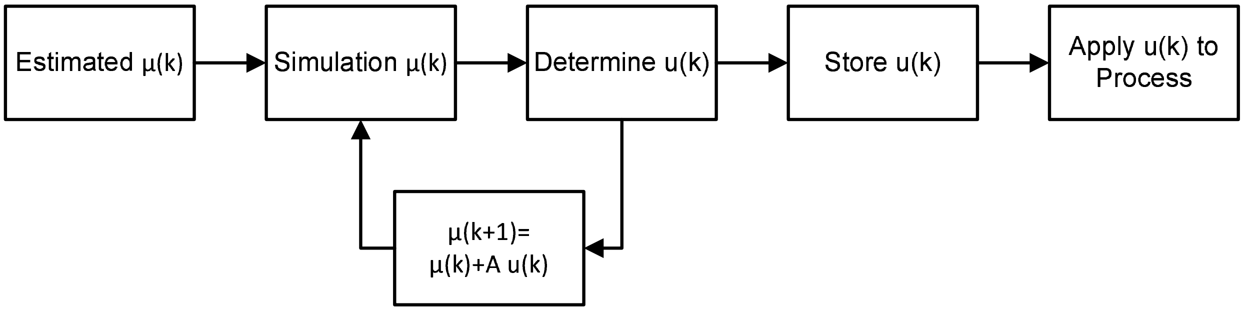


Figure 7.9: Flow chart of the calculation of a control parameter adjustment through a simulation using proportional feedback controllers.

Figure 7.9 shows that the first step in the simulation is to estimate the current process mean. This is achieved by finding the mean of the measured part's CtQs that led to a control adjustment. For example, if two consecutive yellow units signals a control adjustment, the estimate of the current process mean would be calculated by:

$$\bar{x}(k) = \frac{x(k) + x(k-1)}{2} \tag{7.11}$$

This estimate of current process mean, $\bar{x}(k)$, is then used within the simulation as the initial process mean, $\mu(k = 0)$. Proportional feedback controllers are then applied within the simulation environment to determine control parameter adjustments, by:

$$u(k) = G[\mu(k) - T] \tag{7.12}$$

The effects of $u(k)$ on the simulation process mean, $\mu(k)$, is then estimated, by:

$$\mu(k+1) = \mu(k) + A \cdot u(k) \tag{7.13}$$

were, $A$ is this control matrix that links changes in control parameters to changes in the process mean. In direct feedback control, described in the previous section, this physical parameter of the process does not need to be known. To apply simulation feedback control, an estimate of $A$ needs to be made. In practice, this needs to be determined from the machining centre program.

The online simulation continues to make these virtual control parameter adjustments for a fixed number of steps, $l$. The total control parameter adjustment to be applied to the real process is calculated by summing all the virtual control parameter adjustments, by:

$$u(total) = u(k) + u(k+1) + \ldots + u(k+l) \tag{7.14}$$

Once the fixed number of simulation steps, $l$, has been completed, the final control parameter adjustment, $u(total)$ is applied to the real process. The process then continues to make products that are continuously measured until the process is either validated or an alarm is signalled by mSUPA.

The previous example of a process with $\mu(k = 0) = [216, 60, 150]^T$ was re-simulated using simulated feedback control, rather than direct feedback control. In the virtual proportional feedback controller, $G = [0.001, 0.001, 0.001]^T$ was used and there was a fixed number of virtual steps of $l = 1000$. Again, 1000 simulation runs were made to minimise the deterministic effects of the pseudo-random number generation. This produced the results that are plotted in figure 7.10.

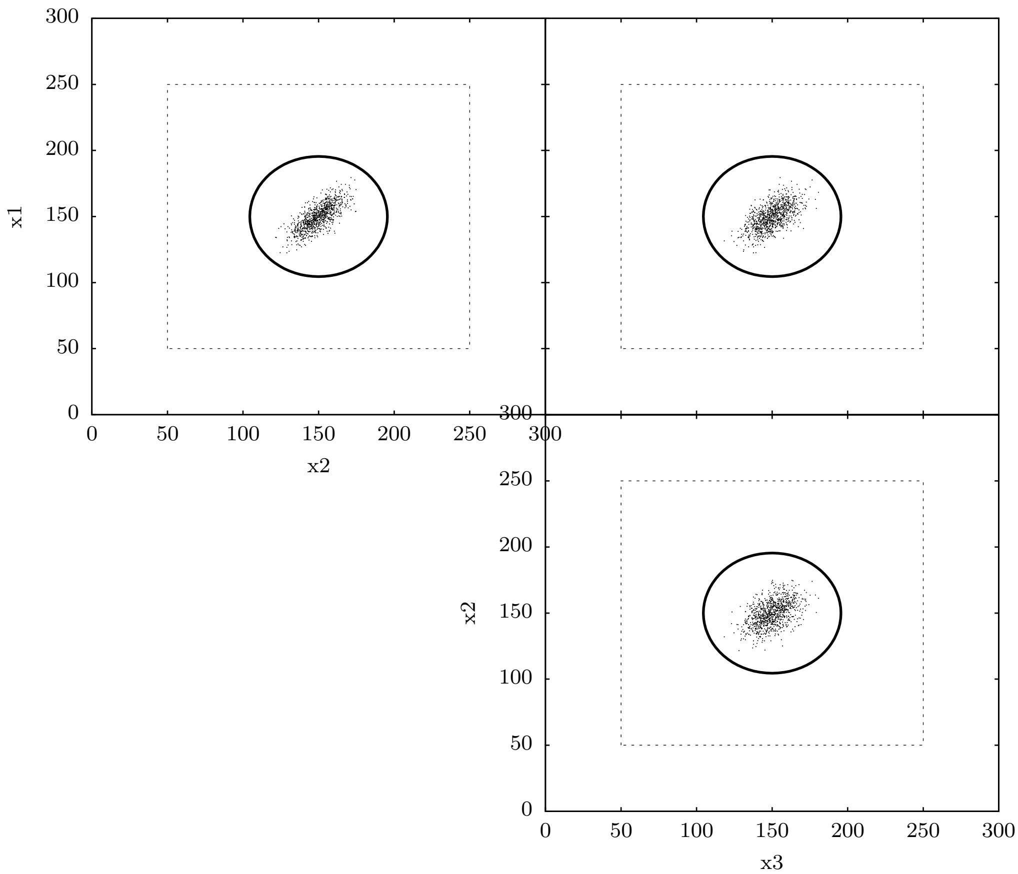


Figure 7.10: Final $\mu$ positions from RtR with simulated proportional feedback control.

In figure 7.10 the final $\mu$ positions from the RtR approach with simulated proportional feedback control are plotted. This plot shows the results are clustered within the green zone around the design target. The cluster of results also forms an ellipse, the shape of which is determined by the covariance of the real process. Because the process contains covariance and the adjustment simulation estimates the current process mean with a small number of units, i.e. one or two, this variance is transferred into the final $\mu$ positions. Further results from this simulation experiment shows that this RtR approach requires an average of 1.199 process adjustments before validation. This required an average of 10.020 products.

These results demonstrate that the use of virtual process control adjustments, derived by proportional feedback controllers, produces more accurate results with final $\mu$ positions clustered around the design target when compared with direct proportional feedback controllers. The virtual process control adjustment cycle allows the proportional feedback controllers to converge on a final real process control adjustment. When a direct proportional feedback control is used, its path towards the design target is interrupted when the process mean is sufficiently within the green zone to be validated. The virtual process control adjustment cycle also significantly reduces both the number of process adjustment, from 174.635 to 1.199, and the number of products collected in measured, from 465.911 to 10.020 in this example.

In all the experiments conducted this far in the chapter there has been the same number of control parameters to adjust as CtQ factors they affect. However, as described in section 6.2, it is common in modern machining centres for there to be fewer control parameters to adjust than there are CtQs they affect. This imbalance causes an issue with reachability, i.e. the proportional feedback controllers are not capable of steering the process towards its design target. This topic is explored in the following section.

## 7.4 Reachability

### 7.4.1 Convergence

It was outlined, in section 6.2, that in practical shop floor operations it was common for complex products to be machined in single processors where there are fewer control parameters than there are CtQ design features, i.e. $n > m$. This means that there are fewer dimensions in $u$ than in $x$; therefore, the system is under-defined.

If the case of $n = 3$ and $m = 2$ is taken such that:

$$A = \begin{bmatrix} 78 & 45 \\ -24 & 93 \\ 25 & -16 \end{bmatrix}. \tag{7.15}$$

The effect of $u$ on $\mu$ can be calculated by expanding the following equation:

$$\mu(k+1) = \mu(k) + A \cdot u(k). \tag{7.16}$$

However, the simulated proportional feedback control is described in the previous section cannot be directly applied in this situation. New proportional feedback control calculations need to be defined in order to calculate values of $u(k)$ when an adjustment is required. An example arrangement that can be used to calculate these values of $u(k)$ is:

$$u_1(k) = G_1[x_1(k) - T_1] + G_3[x_3(k) - T_3].$$
$$u_2(k) = G_2[x_2(k) - T_2] + G_3[x_3(k) - T_3]. \quad (7.17)$$

Using this proportional feedback control arrangement allows the deviation from design target to be considered in all three process dimensions. If the mSUPA based simulated feedback control RtR method, described in the previous section, is applied with balanced gain values of $G = [0.001, 0.001, 0.001]^T$ to a process with a initial process mean of $\mu(k = 0) = [215, 215, 215]^T$, this results in the control actions plotted in figure 7.11.

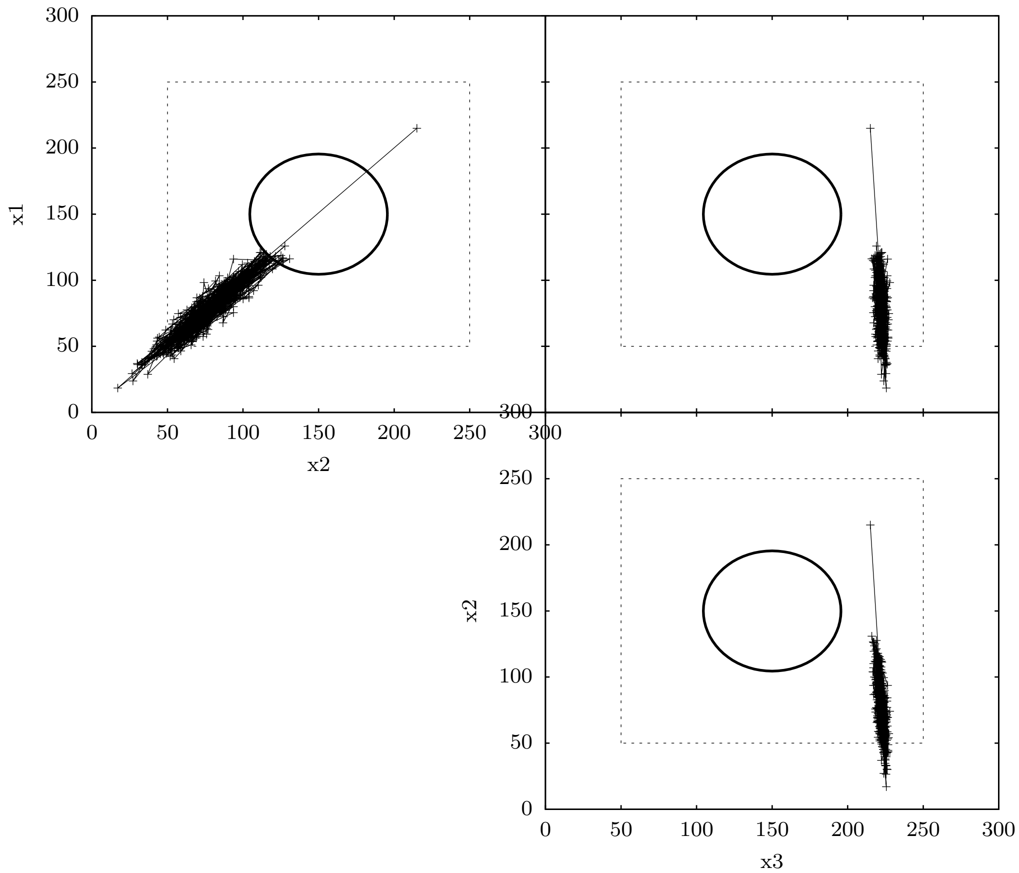


Figure 7.11: $\mu$ positions after adjustments made by the mSUPA based simulated feedback control RtR method.

Figure 7.11 shows the changes in $\mu$ after each adjustment made, which highlights an initial move from $\mu(k = 0) = [215, 215, 215]^T$ to $\mu(k = 1) = [54, 66, 220]^T$, after which $\mu$ wanders around this position. This example demonstrates the reachability problem generated by the imbalance between the number of control parameters and number of CtQ design features.

Due to the lack of control parameters, the process cannot be adjusted directly towards its design target, i.e. the design target is not reachable. In fact, in this case the optimum position that can be achieved by the proportional feedback controllers is not even inside the green zone. This means that mSUPA consistently signals an alarm for another adjustment to be made, which creates the cluster of results around the $\mu(k = 1)$ position. This poses the question, if a design target is not reachable but the process can be maintained within its design tolerances, how can the constant alarm signals from mSUPA be stopped?

### 7.4.2 Adjustable Green Zone Position

To resolve this situation a method of adjusting the position of the green zone is proposed. Given that the proportional feedback controllers are applied virtually, a predicted process mean position can be estimated in this simulation. Therefore, it is proposed that this estimate of the process mean after control parameter adjustments is used as the new centre of the green zone, provided that this does not force the green zone boundary to cross the red zone boundary. This enables a process whose optimum position is not close to the design target to be monitored with the same performance as the mSUPA method described in the previous section monitoring a process whose optimum position is on the design target.

The example process described in this section, can then be re-simulated using this mSUPA with adjustable green zone based simulated feedback control RtR method. In this situation $10,000$ uniformly distributed $\mu(k = 0)$ process positions a simulated to identify which areas of the process space can be adjusted within the design tolerances. The results from the simulation are presented in figures 7.12 and 7.13.

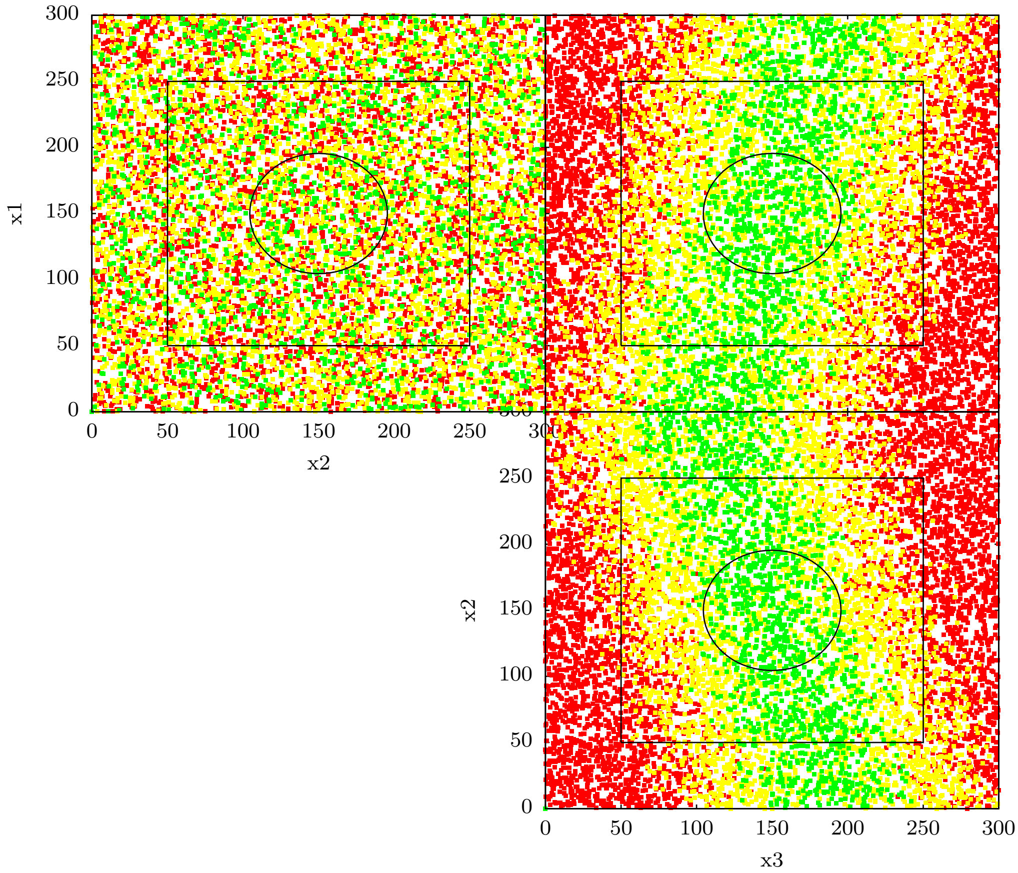


Figure 7.12: Starting $\mu(k=0)$ positions of simulation with $G = [0.001, 0.001, 0.001]^T$.

Figure 7.12 shows the initial $\mu(k=0)$ positions. The points show and are also colour-coded green, yellow and red. This colour coding relates to whether the point falls into the traditional green zone based on design target, or the red and yellow zones after it has been optimised using the mSUPA with adjustable green zone based simulated feedback control RtR method. In effect, this plot is showing reachability bands of the process which are determined by the control matrix, $A$, and the design of the proportional feedback controllers. It can be seen that with respect to $x_3$, there are firm boundaries where the process needs to fall into if it is to be adjusted to a position that is within the design tolerance, this is signified by the green, yellow and red points forming solid regions over the process space. With respect to $x_1$ and $x_2$ the initial starting position does not affect whether the process can be adjusted into a within tolerance position, this is signified by the mottled green, yellow and red points scattered over the process space.

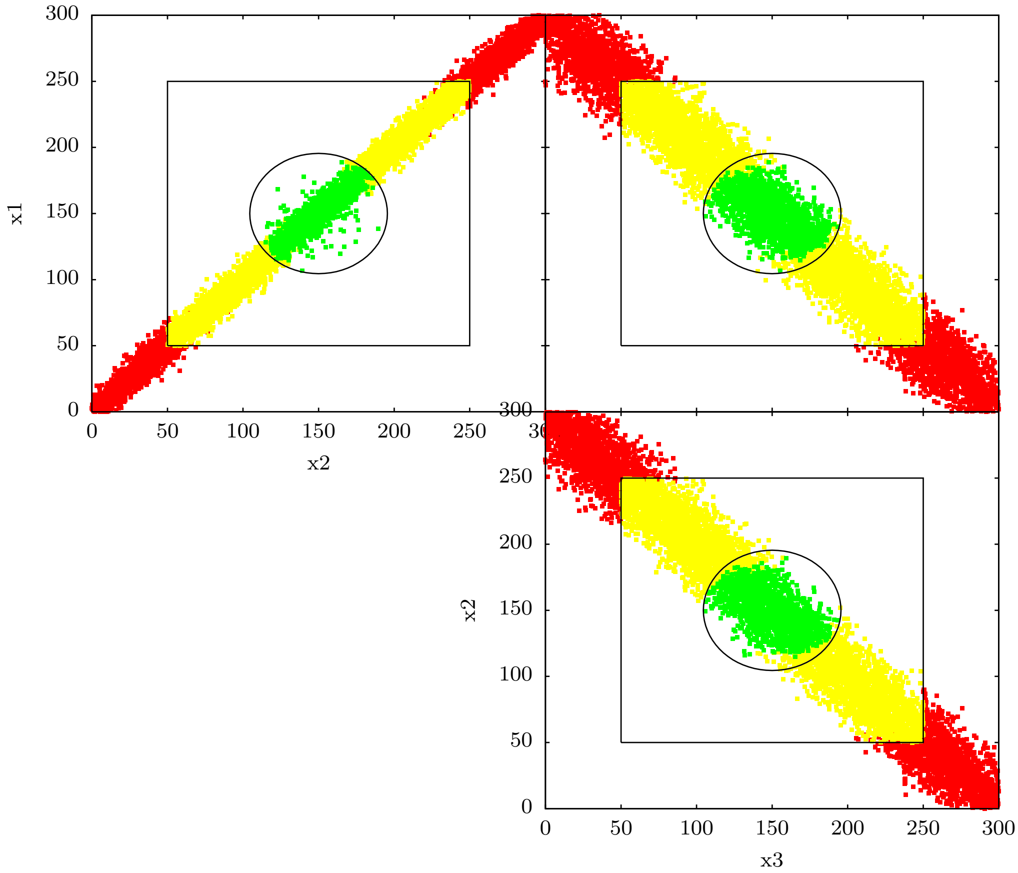


Figure 7.13: Final $\mu$ positions of simulation with $G = [0.001, 0.001, 0.001]^T$.

Figure 7.13 shows the final $\mu$ positions after being optimised using the mSUPA with adjustable green zone based simulated feedback control RtR method. The points are colour-coded green, yellow and red to show whether they fall within the traditional multivariate green, yellow and red zones. This is because a point may be within the green zone on a $x_1 - x_2$ projection, but could be in the yellow zone on the $x_3$ dimension; therefore, the point is yellow. These results show that a process with the characteristics specified in this example can be manipulated into $\mu$ positions that form solid bands in the process space. In this example, the 10,000 starting positions led to 4,133 in a final red zone, 4,305 in a final yellow zone and 1,562 in a final green zone. In order to improve this ratio proportional feedback controller designs were experimented with.

### 7.4.3 Changing Proportional Feedback Controllers

In the next experiment, the $G$ values of the virtual proportional feedback controllers were changed to see the effect on reachability. Hence, the same simulation was run as the previous example but with $G = [0.003, 0.003, 0.001]^T$. This virtual proportion feedback controller places more weight on $\mu_1$ and $\mu_2$ deviation from the design target. The results from these experiments are plotted in figures 7.14 and 7.15.

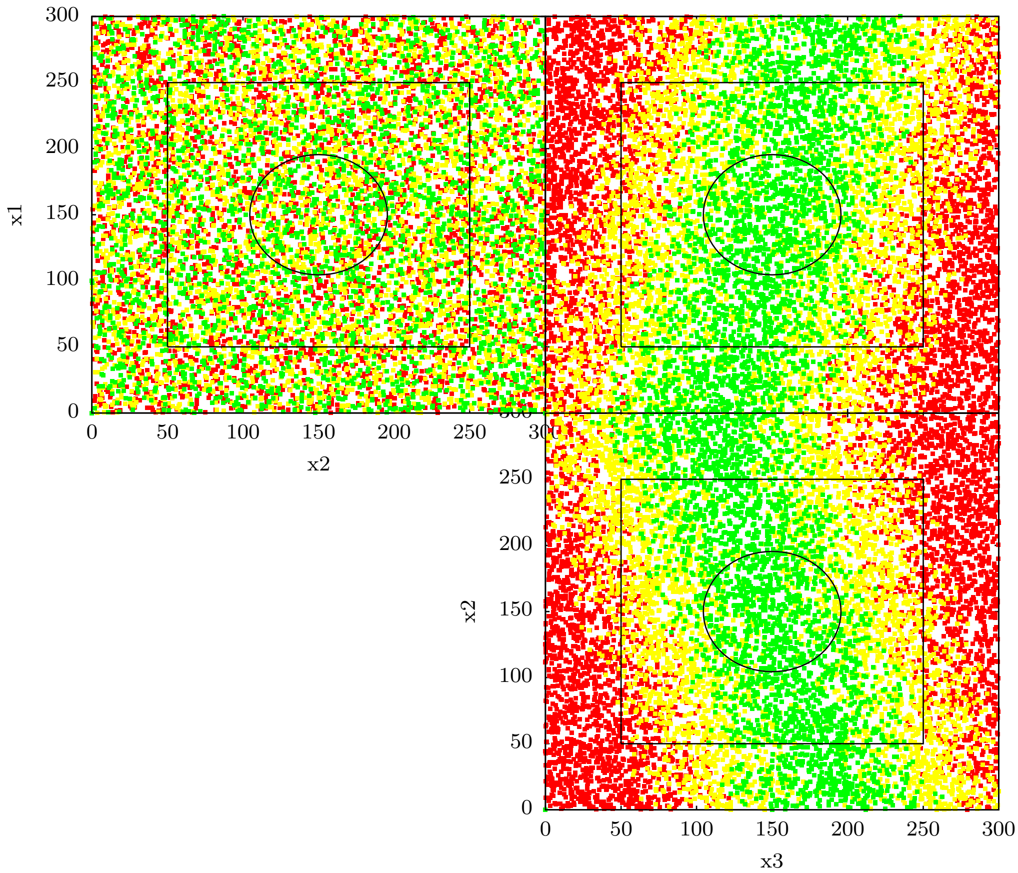


Figure 7.14: Starting $\mu(k = 0)$ positions of simulation with $G = [0.003, 0.003, 0.001]^T$.

Figure 7.14 shows the initial $\mu(k = 0)$ positions, with colour coding relating to whether the point falls into the traditional green zone based on design target, or the red and yellow zones after it has been optimised. When these results are compared to those obtained in figure 7.12, it is seen that both forms similar patterns; there firm boundaries in the $x_3$ planes and no clear boundaries in the $x_1$ and $x_2$ planes demonstrating reachability. The key difference between these two sets of results, shown in figures 7.14 and 7.12, is the much wider green and yellow bands in the $x_3$ plain when feedback controllers with $G = [0.003, 0.003, 0.001]^T$. These wider green and yellow bands indicate that more processes have been directed to a final process position that can be validated by mSUPA as intolerance. Hence, a much greater area of the process space can reach an intolerance position by placing a greater weight on $\mu_1$ and $\mu_2$ deviation from the design target.

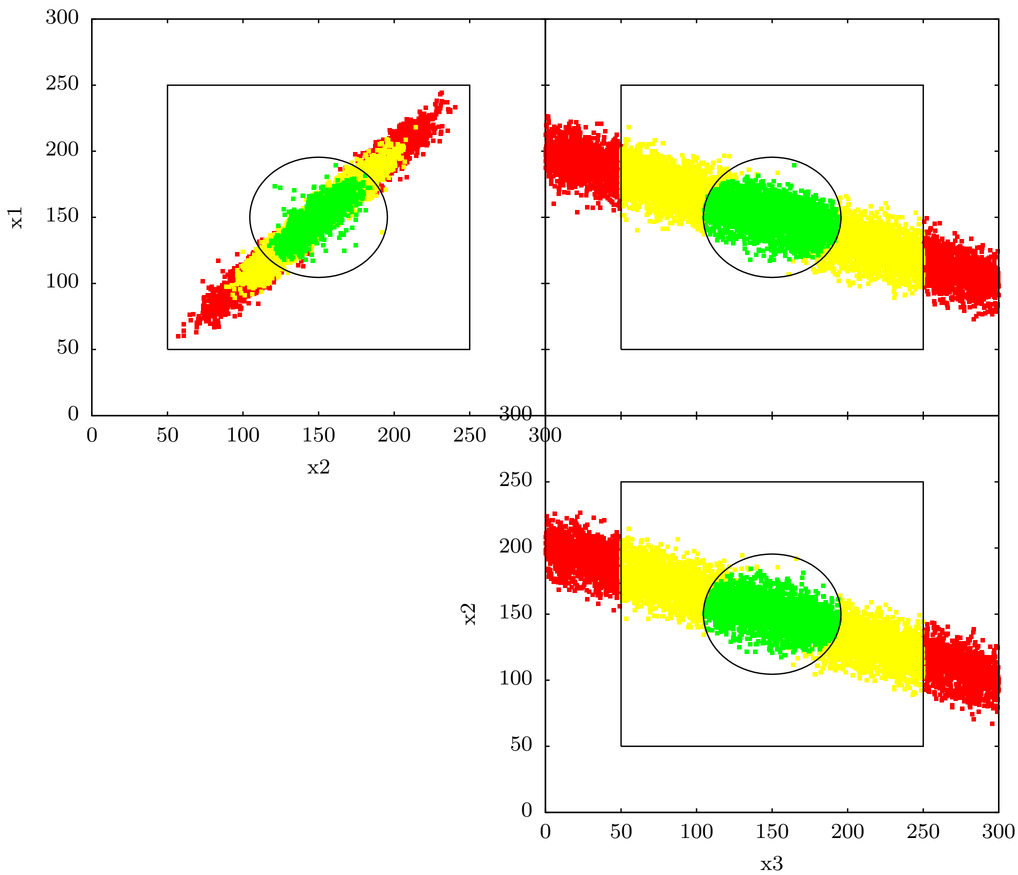


Figure 7.15: Final $\mu$ positions of simulation with $G = [0.003, 0.003, 0.001]^T$.

Figure 7.15 shows the final $\mu$ positions after being optimised with colour-coded green, yellow and red to show whether they fall within the traditional multivariate green, yellow and red zones. These results clearly differ from those presented in figure 7.13; in the $x_1$ and $x_2$ planes all results fall within the processes tolerances, all the results that are red have fallen outside the tolerance in the $x_3$ plane. Further to this, the 10,000 starting positions led to 3,451 in a final red zone, 3,963 in a final yellow zone and 2,586 in a final green zone. This is a significant improvement, with 6,549 processes falling within tolerance as opposed to 5,867.

In addition to this, 2,586 processes fell within the Green zone centred on the design target as opposed to 1,582. This suggests that putting more weight on $\mu_1$ and $\mu_2$ deviation from the design target, has led to more processes falling within tolerance and more accurate centring of the process.

In the next experiment, the $G$ values of the virtual proportional feedback controllers were changed to see the effect on reachability by placing more weight on $\mu_3$ deviation from the design target. Hence, the same simulation was run as the previous example but with $G = [0.001, 0.001, 0.003]^T$. The results from these experiments are plotted in figures 7.16 and 7.17.

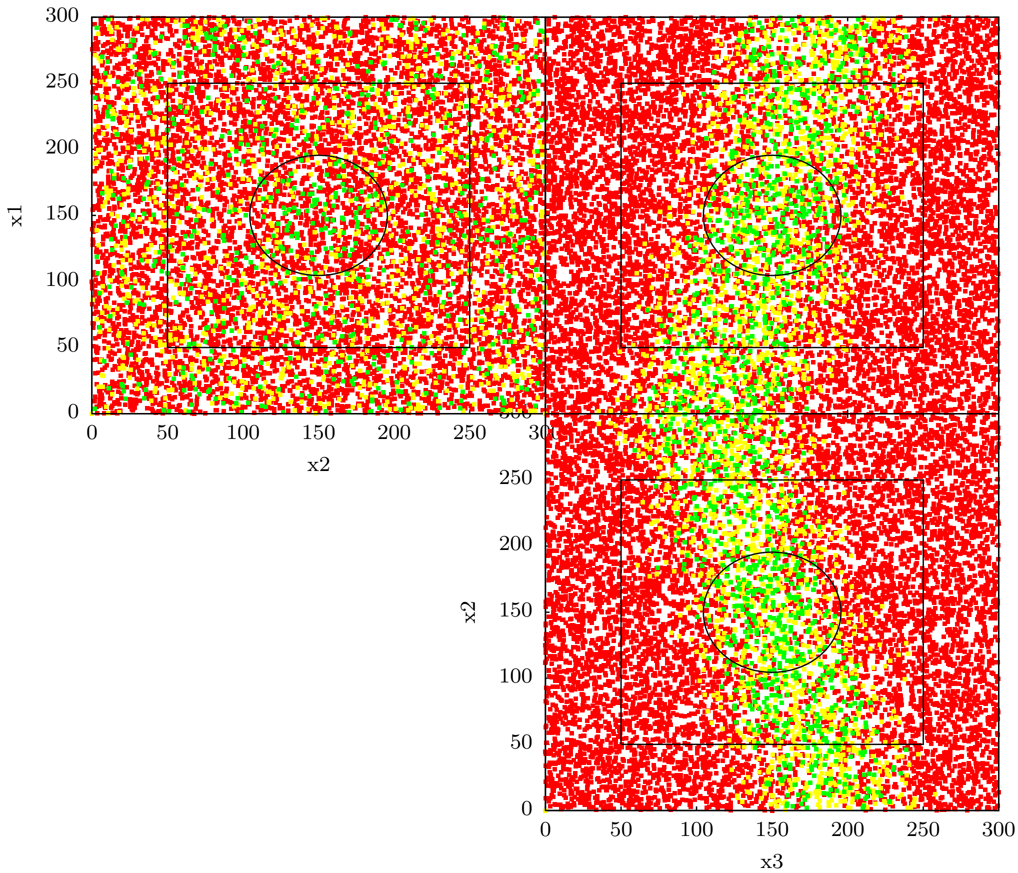


Figure 7.16: Starting $\mu(k = 0)$ positions of simulation with $G = [0.001, 0.001, 0.003]^T$.

Figure 7.16 shows the initial $\mu(k = 0)$ positions, with colour coding relating to whether the point falls into the traditional green zone based on design target, or the red and yellow zones after it has been optimised. When these results are compared to those obtained in figure 7.12 and 7.14, it is seen that in all similar patterns are; there are firm boundaries in the $x_3$ planes and no clear boundaries in the $x_1$ and $x_2$ planes demonstrating reachability. The key difference between these sets of results is the much narrower green and yellow bands in the $x_3$ plain when feedback controllers with $G = [0.003, 0.003, 0.001]^T$ are deployed. These narrower green and yellow bands indicate that fewer processes have been directed to a final process position that can be validated by mSUPA as intolerance. Hence, a much smaller area of the process space can reach an intolerance position by placing a greater weight on $\mu_3$ deviation from the design target.

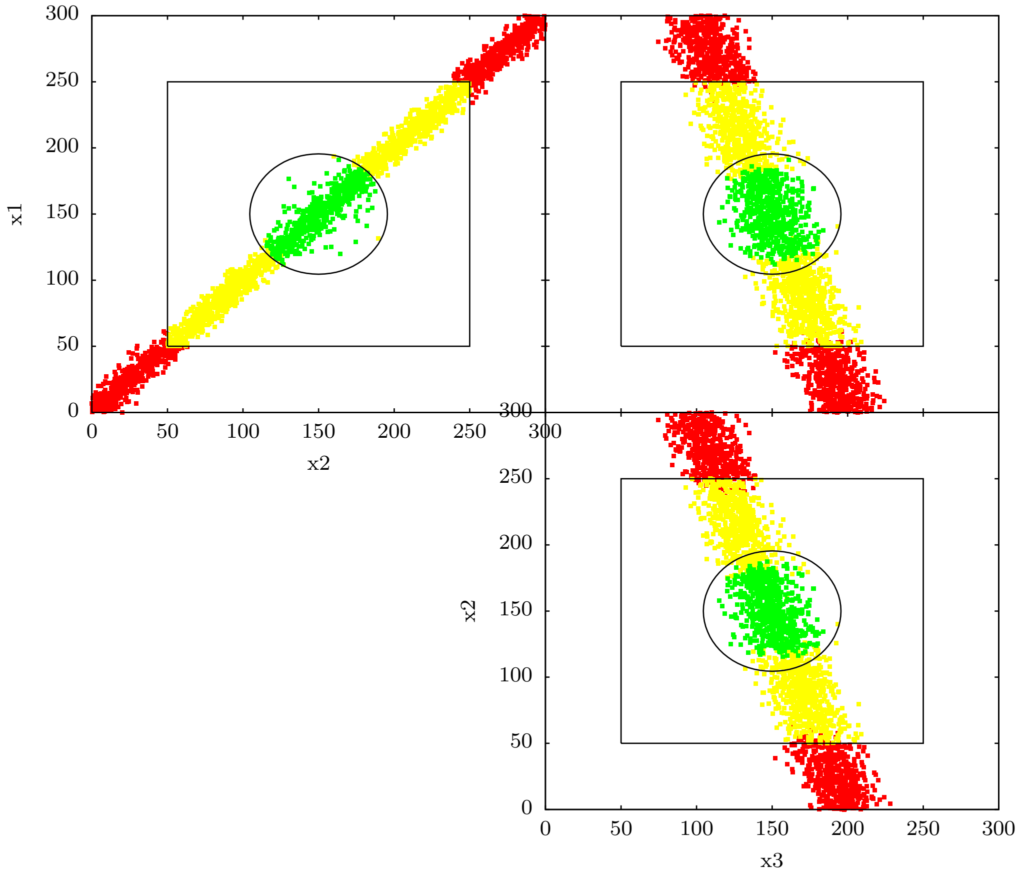


Figure 7.17: Starting $\mu(k = 0)$ positions of simulation with $G = [0.001, 0.001, 0.003]^T$.

Figure 7.17 shows the final $\mu$ positions after being optimised with colour-coded green, yellow and red to show whether they fall within the traditional multivariate green, yellow and red zones. These results clearly differ from those presented in figure 7.13; in the $x_3$ plane all results fall within the process tolerance, all the results that are red have fallen outside the tolerance in the $x_3$ plane. Further to this, the 10,000 starting positions led to 9,938 in a final red zone, 3 in a final yellow zone and 59 in a final green zone. This is a significant deterioration, with 62 processes falling within tolerance as opposed to 5,867. In addition to this, 59 processes fell within the green zone centred on the design target as opposed to 1,582. This suggests that putting more weight on $\mu_3$ deviation from the design target, has led to fewer processes falling within tolerance and less accurate centring of the process.

This section has clearly highlighted the issue of reachability when a process has fewer control parameters than CtQ design features. A method of adjusting the green zone of an mSUPA chart has been proposed to stop false alarm signals once a process has been optimised. Also, it has been shown that the design of the virtual proportional feedback controller has a significant effect on reachability. To use this method in practice would require a high level of tuning of the $G$ values, which may not be possible in a low-volume process. Proportional feedback controllers were initially used because they provided an approach to directly control the system. However, because these calculations are now made in the virtual space this provides an opportunity to use numerical optimisation techniques rather than feedback controllers. Therefore, the next section proposes a RtR methodology that is based on mSUPA with a location adjustable green zone and a virtual numerical optimisation approach to calculating the size of control parameter adjustments.

## 7.5 Proposed Methodology

### 7.5.1 Methodology

In sections 7.3 and 7.4 an mSUPA with position adjustable green zone based simulated proportional feedback control RtR method was described and tested. An issue with the approach was the difficulty in forming proportional feedback controller calculations and tuning the gain values. However, by simulating process adjustment there is an opportunity to use other optimisation techniques in place of the proportional feedback controllers. This section explores the potential of applying a local search optimisation technique known as hill climbing to the adjustment of control parameters.

In essence the hill climbing method defines an objective function to maximise or minimise. From the current position, variables are adjusted iteratively to search the local space around the current position for an improvement in the objective function. This process is continued until a position is found with no improvements in the objective function in the local space. If the approach is applied to optimise the example problem of a process with 3 CtQ features and 2 control parameters, the objective function $f(\mu(k+1))$ is define as:

$$\begin{aligned} \underset{\mu}{\text{minimize}} \quad & f(\mu(k+1)) = \sqrt{|\mu_1(k+1) - T_1|^2 + |\mu_2(k+1) - T_3|^2 + |\mu_3(k+1) - T_3|^2}, \\ \text{subject to} \quad & \mu(k+1) = \mu(k) + A \cdot u(k). \end{aligned} \tag{7.18}$$

Therefore, $f(\mu(k+1))$ is minimised with respect to the design target, $T$, by adjusting the control parameters, $u$. This optimisation algorithm is then placed in the mSUPA with position adjustable green zone based simulated control RtR method as in figure 7.18.

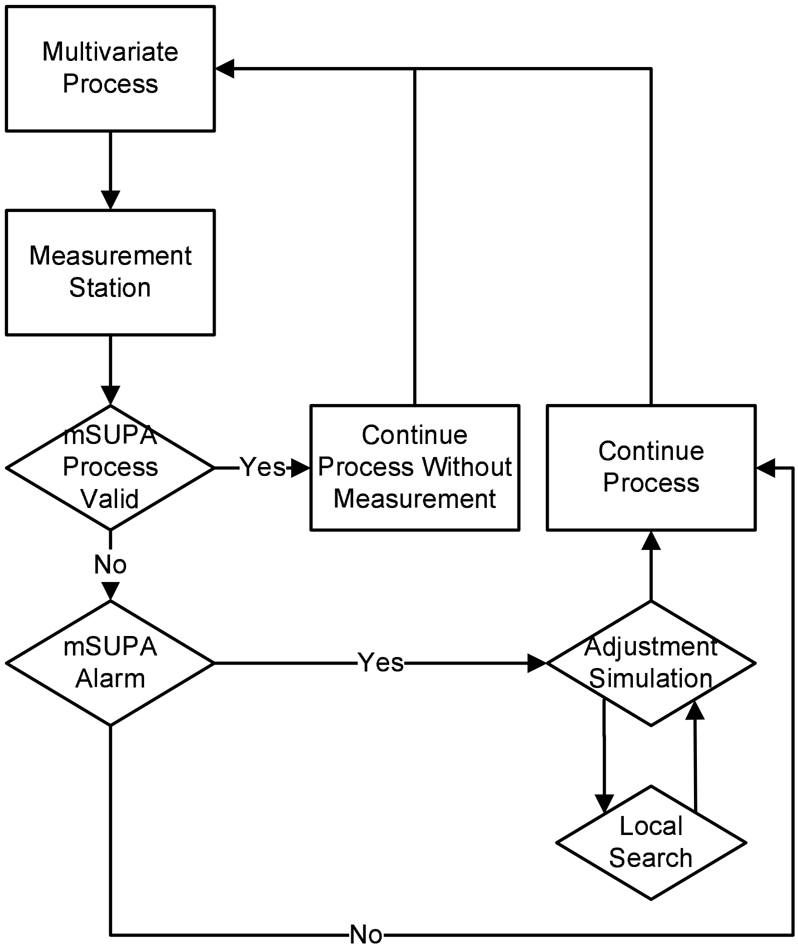


Figure 7.18: Flow chart of mSUPA based simulated hill climbing adjustment RtR method.

The remainder of this section uses examples to demonstrate the effectiveness of this approach.

### 7.5.2 Local Search Optimisation Example

In this example, the same process defined in the previous section is simulated and controlled with the mSUPA plus hill climbing RtR method. In this situation $10,000$ uniformly distributed $\mu(k=0)$ process positions are simulated to identify which areas of the process space can be adjusted within the design tolerances. The results from the simulation are presented in figures 7.19 and 7.20.

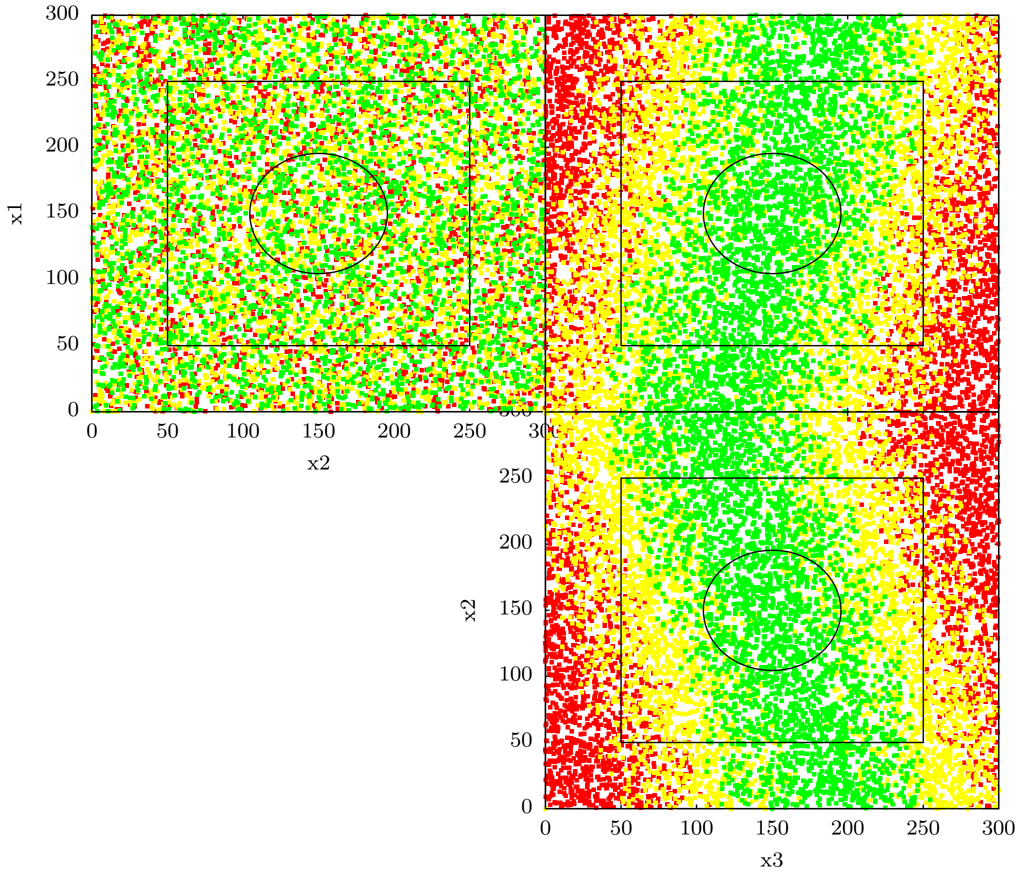


Figure 7.19: Starting $\mu(k = 0)$ positions of simulation controlled by the mSUPA plus hill climbing RtR method.

Figure 7.19 shows the initial $\mu(k = 0)$ positions with there final colour coding. It can be seen that with respect to $x_3$, there are firm boundaries where the process needs to fall into if it is to be adjusted to a position that is within the design tolerance, this is signified by the green, yellow and red points forming solid regions over the process space. With respect to $x_1$ and $x_2$ the initial starting position does not affect whether the process can be adjusted into a within tolerance position, this is signified by the mottled green, yellow and red points scattered over the process space. This effect is similar to the results shown in figures 7.12, 7.14 and 7.16 when virtual proportional feedback controllers were used. However, there is a difference in the width of the green and yellow bands in the $x_3$ plane, in the hill climbing case these bands are wider than any of the virtual proportional feedback controllers. This suggests the hill climbing approach steers a greater number of processes towards an in-tolerance position.

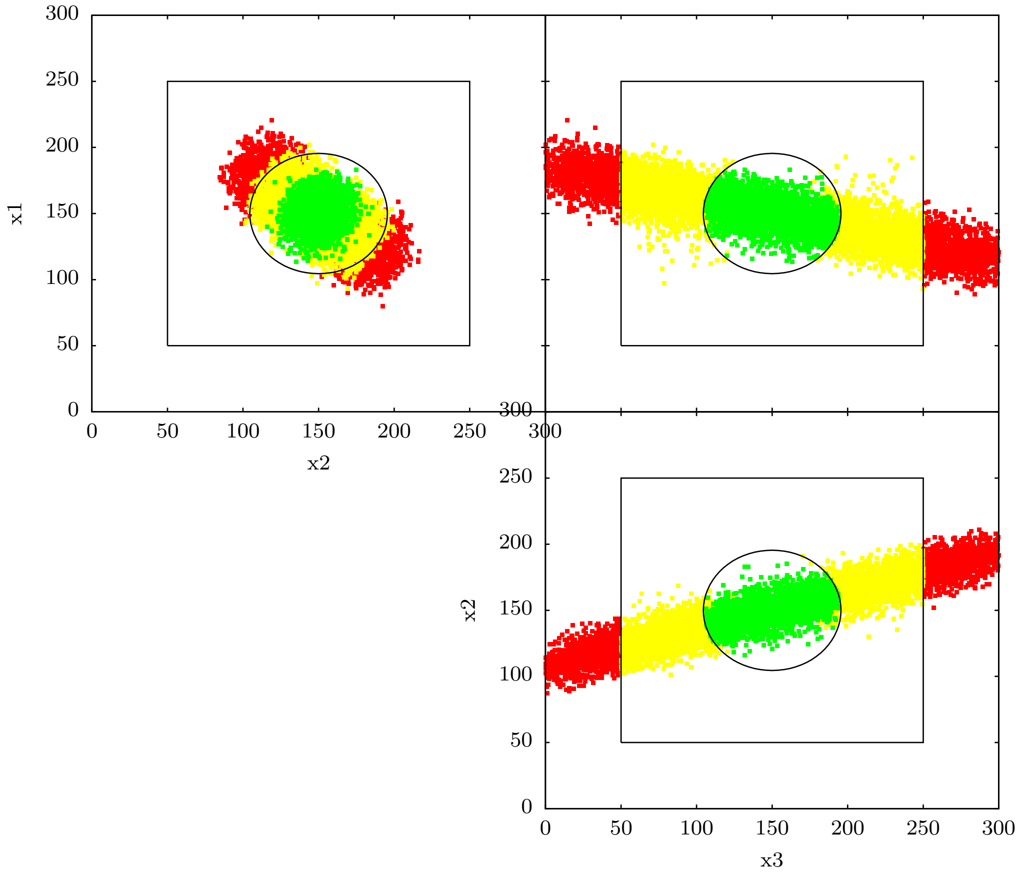


Figure 7.20: Final $\mu(k)$ positions of simulation controlled by the mSUPA plus hill climbing RtR method.

Figure 7.20 shows the final $\mu$ positions after being optimised with there final colour-coding. These results clearly differ from those presented in figures 7.13, 7.15 and 7.17; in the $x_1$ and $x_2$ planes all results fall within the processes tolerances forming a tight elliptical cluster around the central green zone; all the results that are red have fallen outside the tolerance in the $x_3$ plane. Further to this, the 10,000 starting positions led to 2,546 in a final red zone, 4,339 in a final yellow zone and 3,115 in a final green zone. This is a significant improvement, with 7,454 processes falling within tolerance as opposed to 6,549 for the best performing proportional feedback controller. In addition to this, 3,115 processes fell within the green zone centred on the design target as opposed to 2,586 for the best performing proportional feedback controller. These improved results have been achieve without the need for a process specific proportional controller to be designed and tuned. In fact the hill climbing approach is generic and should, therefore, be scalable to fit a process with any number CtQ features and control parameters.

### 7.5.3 Benchmark

#### 7.5.3.1 First-Off and Primary Feature Correction

In the previous section a RtR method that combines mSUPA and hill climbing optimisation was tested in a simulation environment. In order to give these results context, this section provides a benchmark case for comparison and future research can then reference. The benchmark case in this section uses the first-off approach to process control, as this method reflects an approach currently used in industry and has been simulated in chapter 6. In conjunction with first-off, this benchmark uses a primary feature correction approach to apply adjustment feedback. This combined approach of first-off and primary feature correction only measures the first sample, if a feature is outside its tolerance the primary feature correction approach is used to adjust the process but no other samples are measured. The primary feature correction approach works by: firstly, determining the number of control parameters, $m$; then, the control parameter, $u(k)$ are adjusted to steer the first $m$ of $n$ measured critical features, $x(k)$, to their design target, $T$. For example, if there are $m = 2$ control parameters and there are $n = 3$ critical features, the size of the adjustment to $u(k)$ is calculated by solving:

$$\begin{aligned} T_1 - x_1(k) &= a_{11} \cdot u_1(k) + a_{12} \cdot u_2(k) \\ T_2 - x_2(k) &= a_{21} \cdot u_1(k) + a_{22} \cdot u_2(k) \end{aligned} \tag{7.19}$$

then the final effects of the control parameters on the next unit are calculated by:

$$\begin{aligned} x_1(k+1) &= x_1(k) + a_{11} \cdot u_1(k) + a_{12} \cdot u_2(k) \\ x_2(k+1) &= x_2(k) + a_{21} \cdot u_1(k) + a_{22} \cdot u_2(k) \\ x_3(k+1) &= x_3(k) + a_{31} \cdot u_1(k) + a_{32} \cdot u_2(k) \end{aligned} \tag{7.20}$$

where $a_{ij}$ of elements of the $A$ matrix outlined in equation (7.16). The remainder of this section uses examples to demonstrate the effectiveness of this approach.

#### 7.5.3.2 Benchmark Simulation Example

In this example, the same process defined in the previous section is simulated and controlled with the first-off and primary feature correction. In this situation $10,000$ uniformly distributed $\mu(k = 0)$ process positions are simulated to identify which areas of the process space can be adjusted within the design tolerances. The results from the simulation are presented in figures 7.21 and 7.22. The same $A$ matrix is used in this example as was used in the mSUPA and hill climbing example, thus, providing a direct comparison.

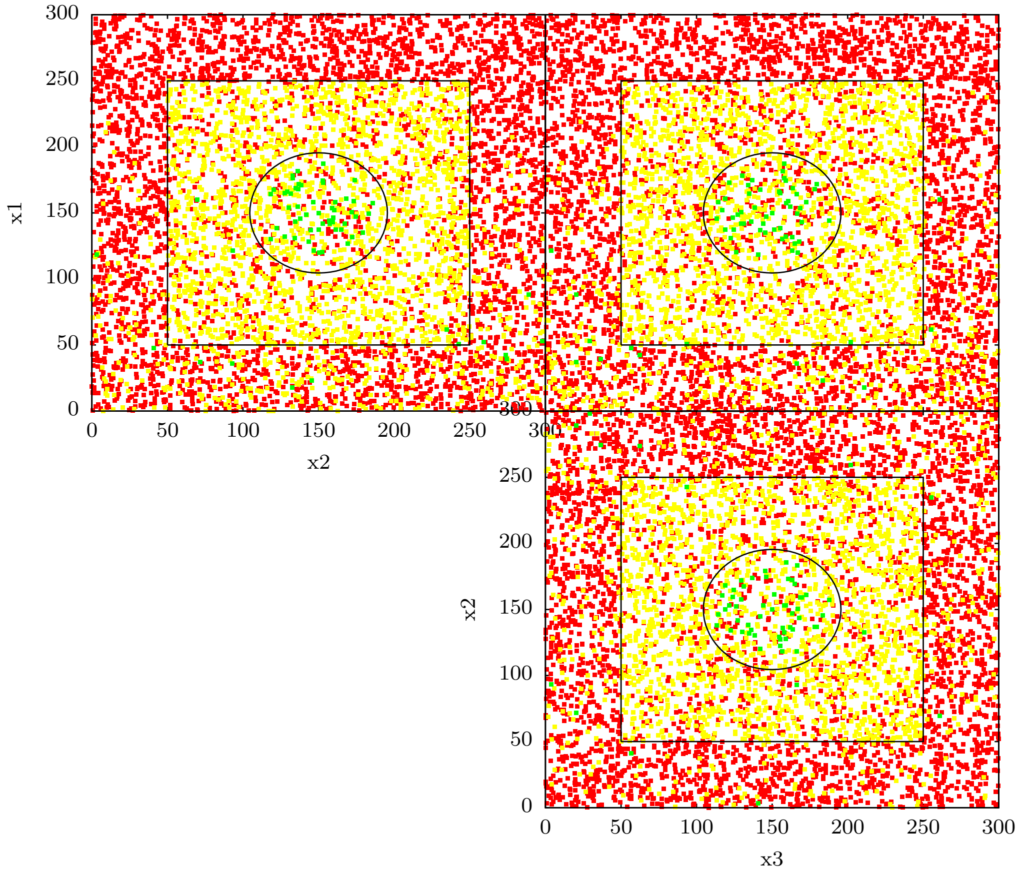


Figure 7.21: Starting $\mu(k = 0)$ positions of simulation controlled by the first-off and primary feature correction benchmark method.

Figure 7.21 shows the initial $\mu(k = 0)$ positions with there final colour coding. This figure shows that there are very few points that start outside of the tolerance band that finish as either a yellow or green process. Hence, it is demonstrating that few processes have their mean position improved globally using the first-off and primary feature adjustment method.

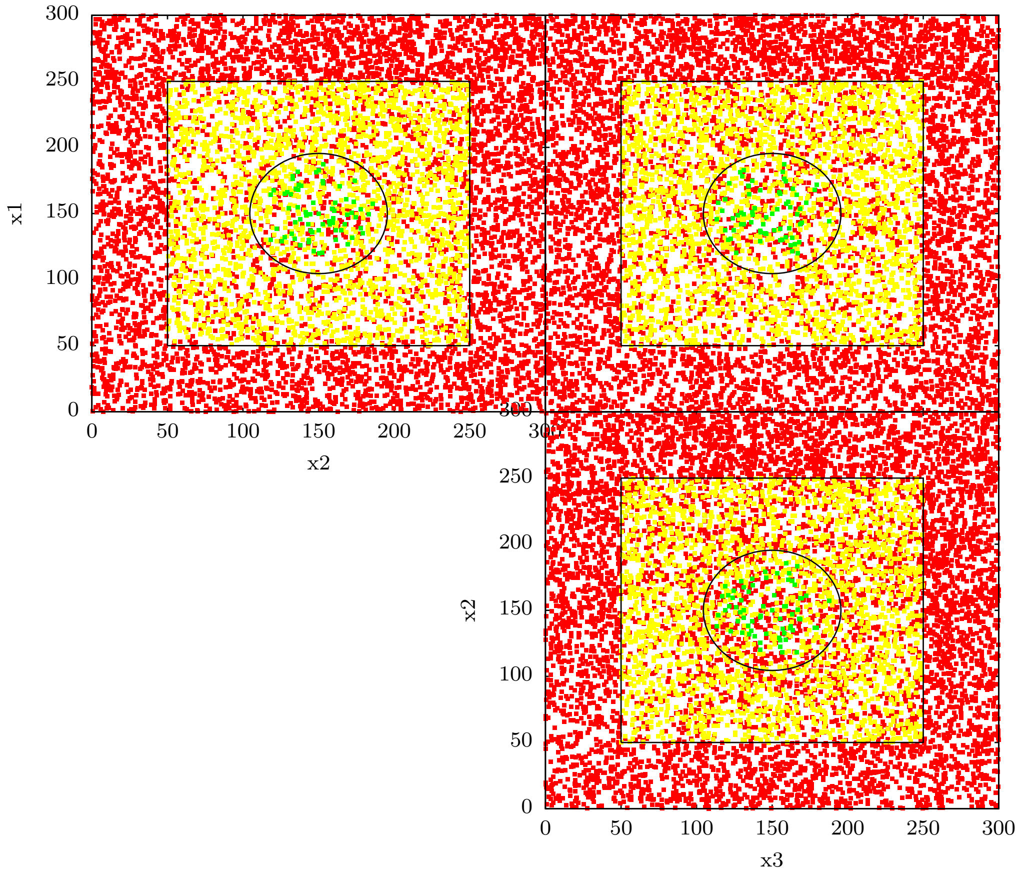


Figure 7.22: Final $\mu(k)$ positions of simulation controlled by the first-off and primary feature correction benchmark method.

Figure 7.22 shows the final $\mu$ positions after being optimised with there final colour-coding. These results do not differ a great deal from the initial positions shown in figure 7.21. Further to this, the 10,000 starting positions led to 7,741 in a final red zone, 2,189 in a final yellow zone and 68 in a final green zone. This performance is poor when compared with the mSUPA and hill climbing approach, with 2,257 processes falling within tolerance after application of the first-off and primary feature correction method as opposed to 7,454 for the mSUPA and hill climbing approach. In addition to this, 68processes fell within the green zone centred on the design target as opposed to 3,115 for the mSUPA and hill climbing approach.

### 7.5.4 Higher Order Example

This test, assesses the scalability of the mSUPA with position adjustable green zone based simulated hill climb control RtR method against a new simulated process with $n = 5$ CtQ factors and $m = 2$ control parameters. The process maintains the multivariate normal distribution described in equation 7.7, but with the following covariance matrix:

$$\Sigma = \begin{bmatrix} 100 & 50 & 0 & 50 & 0 \\ 50 & 100 & 50 & 40 & 10 \\ 0 & 50 & 100 & 0 & 40 \\ 50 & 40 & 0 & 100 & 30 \\ 0 & 10 & 40 & 30 & 100 \end{bmatrix}. \tag{7.21}$$

This $\Sigma$ value describes the correlations between the five processed CtQ features. The next feature that is defined is the $A$ matrix, which links changes in the control parameters, $u$, to their affect on CtQ factors mean position, $\mu$, as follows:

$$A = \begin{bmatrix} 50 & -50 \\ 0 & 100 \\ 100 & 0 \\ 56 & 67 \\ 0 & 75 \end{bmatrix}. \tag{7.22}$$

In this situation $10,000$ uniformly distributed $\mu(k = 0)$ process positions are simulated. The results from the simulation are presented in figures 7.23 and 7.24.

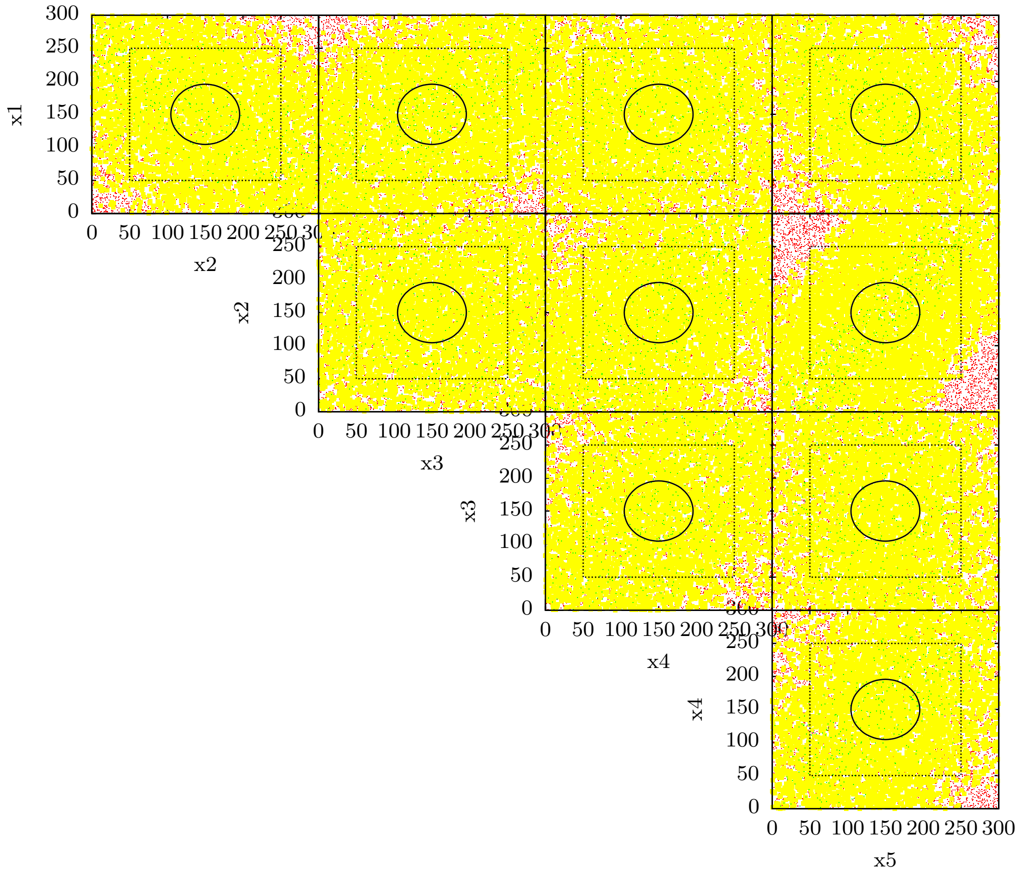


Figure 7.23: Starting $\mu(k = 0)$ positions of simulation controlled by the mSUPA plus hill climbing RtR method.

Figure 7.23 shows the initial $\mu(k = 0)$ positions with there final colour coding. Unlike the previous examples, of a three dimension process, this five dimension process does not have clear cut green/yellow/red boundaries. The $x_2$-$x_5$ projection has the clearest boundaries with a diagonal green band. However, despite the imbalance between $n$ and $m$, there is a large area of yellow points on all planes. This suggests that hill climbing optimisation is able to steer the process to a globally in-tolerance position.

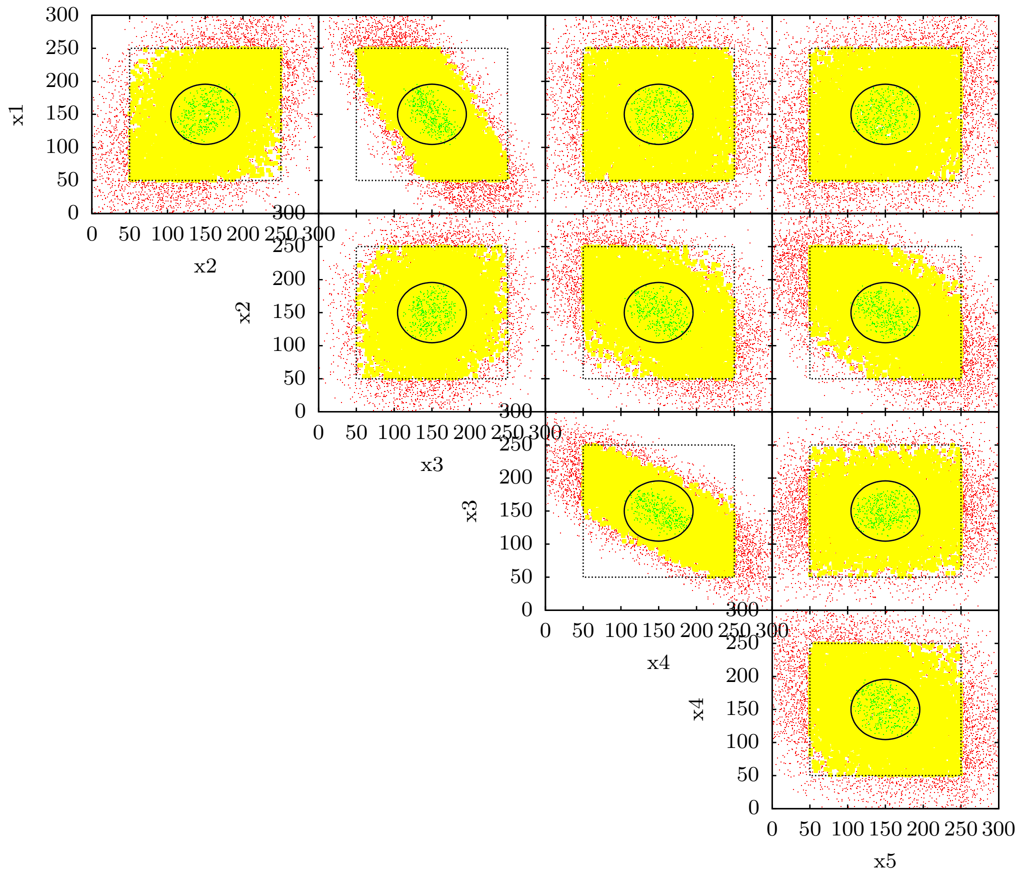


Figure 7.24: Final $\mu$ positions of simulation controlled by the mSUPA plus hill climbing RtR method.

Figure 7.24 shows the final $\mu$ positions after being optimised with colour-coding. Of the 10,000 starting positions 8,733 where in a red zone, 1,252 where in a yellow zone and 14 where in a green zone. This led to 4,887 in a final red zone, 4,723 in a final yellow zone and 389 in a final green zone. This confirms the previous observation that hill climbing optimisation is able to steer the process to a globally intolerance position despite the imbalance between $n$ and $m$. This experiment highlights that the mSUPA with position adjustable green zone based simulated hill climb control RtR method is scalable and steers processes to an optimum position despite an imbalance in $n$ and $m$.

## 7.6 Summary

This chapter set out to explore the feasibility of providing automated information to operators to adjust the process control parameters; in order, to optimise the CtQ features on manufactured products. The study was undertaken due to the prevalence of operators currently using rule-of-thumb approaches to adjusting process control parameters. The importance of this study is becoming ever more critical as manufacturing processes produce ever more complex products; resulting in processes that have considerably fewer control parameters to adjust than CtQ features to optimise.

An mSUPA based RtR method was outlined as a basic strategy to signal when an adjustment was needed. Within this method different approaches to automating the calculation of control parameter adjustments where tested. Initially a direct proportional feedback controller was tested due to its use in other RtR methods described in the literature. This approach was shown to drive processes towards their target when the gain, $G$, of the controller was sufficiently small. However, the smaller the $G$ value used the greater the number of adjustments that was required, which is undesirable.

To reduce the number of adjustment an online simulation using a virtual proportional feedback controller was developed and tested. Results from experimentation with this virtual proportional feedback controller highlighted that it was able to drive processes closer to their design target with fewer real-world adjustments required when compared to direct proportional feedback control. However, the proportional feedback control method had an issue with reachability when the process had fewer control parameters than CtQ design features.

To overcome this reachability issue a numerical optimisation technique known as hill climbing was used in the online simulation in place of the virtual proportional feedback controller. The hill climbing technique performed better than the virtual proportional feedback controller in the situation when there are fewer control parameters than CtQ design features. It consistently drove processes as close to the CtQ design target as feasibly possible. The hill climbing technique showed that when it was applied in the same circumstances as the virtual proportional feedback controller a greater number of processes ended up inside the tolerance boundary.

# Chapter 8

# Conclusion



## 8.1 Introduction

This research set out to explore the available process improvement methodologies and tools suitable for application to low volume manufacturing processes. The thesis aimed to derive statistically valid process improvement tools for use by industry practitioners. This led to the development and testing of tools to specifically assist a practitioner to efficiently Measure, Analyse and Control a quality problem. The research was conducted under the consideration of Advanced Manufacturing Technology (AMT); whereby, a single process is producing products in low-volumes with: high-variety, high-precision, high-value and high-complexity. A further consideration was the practitioners requirements; although all the tools developed would be underpinned by statistical analysis, they are intended for use by process operators who do not have a background in statistics. Therefore, this research asked the following questions:

**RQ1** *When improving low-volume processes, what is the relevant process improvement framework to structure the deployment of statistical tools?*

**RQ2** *Within a proposed framework, how can the measurement and analysis of a chronic quality problem be statistically validated in a low-volume process?*

**RQ3** *What is required to provide a practical and statistically valid approach for process control in a low-volume process?*

**RQ4** *How can a low-volume process control tool be extended to fit multivariate processes?*

**RQ5** *When a low-volume multivariate process control tool signals the process is off-target, how can the calculation of control parameter adjustments be automated?*

## 8.2 Findings

The main findings of this research are chapter specific and were summarised within their respective chapters. However, these results are used here to answer the main research questions:

**RQ1** When improving low-volume processes, what is the relevant process improvement framework to structure the deployment of statistical tools?

This topic was addressed in chapter 2. It was found that using a blend of two seemingly separate process improvement methodologies, Six Sigma and the Shainin System, allowed a rigourous framework to be formed that guides practitioners to a solution to a quality problem. The resulting framework, known as **Hybrid Six Sigma** and outlined in figure 2.3, uses the DMAIC cycle from Six Sigma, that is prevalent in industry, as its backbone. Finding an approach that already has roots in industry was an important result when practitioner requirements are considered, because it eases the transfer of the framework into practice.

A key issue of the DMAIC method was in the *'exploration'* of a quality problem at the Analyse phase; an issue that is exacerbated in the context of low-volume manufacturing problems. Specifically, it was the application of statistically valid tools to problems that contained a large number of potential causes of process variation that challenged practitioners. Typically, a statistical test to identify the main causes of process variation would be deployed. However, in the presence of a large number of potential causes this requires a sample containing a large number of products, which are not always available and can be cost prohibitive in low-volume, high-value processes. Therefore, practitioners of these low-volume, high-value processes had resorted to subjective, non-statistical analysis.

To address the problem of subjective analysis, the Hybrid methodology incorporates the principle of *'narrowing down'* from the Shainin System to strengthen the exploration of a quality problem. This principle uses a series of stratified experiments, which require smaller sample sizes of product to be tested, to rule out potential causes of process variation that are having little or no effect down to the main causes.

**RQ2** Within a proposed framework, how can the measurement and analysis of a chronic quality problem be statistically validated in a low-volume process?

This topic is addressed by chapters 3 and 4. By answering **RQ1**, a process improvement methodology was identified to guide practitioners through the steps required to solve quality problems. This Hybrid Six Sigma framework aimed to improve the exploration of a quality problem; which, occurs at the bridge of the Measure and Analyse phases of the DMAIC cycle. To answer **RQ2**, this research examined: what statistical metrics are required at the Measure and Analyse phases? what tools from the Shainin System are required to implement the narrowing down approach? and, how can these analysis methods be deployed in a low-volume process?

It was identified that there were two generally required statistical metrics from the Measure and Analyse phases: Gage R&R, to validate the performance of the measurement system, and process capability, to asses the current performance of the process. It was also identified that these phases can be supported by the Shainin System toolkit by: the Isoplot, to validate the measurement system, and the multi-vari study, a stratified experiment to identify the time scale of process variation and starting the narrowing down analysis. Further to this, the PROcess VAriation Diagnosis Tool (PROVADT) method was identified as a sampling strategy capable of collecting and measuring products that allows application of all four of the aforemention analysis tools. This is achieved in as few as 20 products and a total of 60 measurements; therefore, PROVADT lends itself to low-volume application.

An Issue with the PROVADT method as it stood, was it confounded some variation sources in the measurement system analysis; in order, to minimise the sample size of products tested. Chapter 4 developed an **enhanced PROVADT** method, which, separated the confounding variation sources by maintaining a minimum sampling size of 20 products, but being measured a total of 80 times. This sample size maintained enhanced PROVADT's application in low-volume applications. This research was novel as it provides practitioners with a means of apply four essential process improvement tools in one sample; rather than applying them sequentially, as is the case traditionally. The traditional approach, ultimately, drives up the number of products tested and the number of measurements made, which places time and financial pressures on the analysis particularly in low-volume applications. The enhanced PROVADT method also delivered positive results in two industrial case studies in short time-frames.

The remainder of the research was focused in the Control phase of the Hybrid Six Sigma methodology. Specifically, how to control low-volume, high-variety processes, which, are classified as *'set-up dominant'* in control texts. In set-up dominant processes, it is impossible to derive the instantaneous statistical performance. Therefore, classic SPC is impossible to apply in its true form; leading practitioners, such as process operators, to manage their equipment with a subjective rule-of-thumb approach. This situation provided the next three research questions:

**RQ3** What is required to provide a practical and statistically valid approach for process control in a low-volume process?

This question is addressed by chapters 3 and 5 and is a critical issue for set-up dominant processes as ever higher precision requirements are demanded. This research generated a new non-parametric control method, known as Set-Up Process Algorithm (**SUPA**), which although underpinned by statistical theory, provided practitioners with a traffic-light chart and simple set of rules. The clarity of the traffic-light chart, which intuitively links control boundaries to tolerance limits, was a critical feature to aid practitioners understanding.

The performance of the SUPA tool was compared with other traditional SPC tools in a low-volume application through discrete-event simulation. A novelty in this research was the use of discrete-event simulation to asses the performance of a control tool, not only up to the point of a control action, but beyond it until a final process valid or invalid decision is made. These simulations identified that SUPA and Small-Batch $\bar{X}R$ Chart (SB$\bar{X}R$) were the best performing control tools against set-up processes with both Gaussian and non-Gaussian distributions. However, SB$\bar{X}R$ had drawbacks: it required an estimate of process performance to establish control limits, this is not always possible in low-volume applications; its control limits also change after each sampled product and two control charts need to be maintained, which is confusing when compared to the clarity of non-parametric approaches such as Pre-Control or SUPA.

**RQ4** How can a low-volume process control tool be extended to fit multivariate processes?

This topic is addressed by chapters 3 and 6 and is a key issue for set-up dominant processes as more high-complex products are manufactured in single processes. As products become more complex they contain more Critical-to-Quality (CtQ) features, this means the associated process needs to be controlled in each CtQ, this raises **RQ4**. One approach would be to monitor a control chart for each feature, but, it was shown in a case study, this becomes less and less practical as the number of CtQs increases. In response, this research looked to extend SUPA to provide a single chart covering all CtQs, resulting in the multivariate SUPA (**mSUPA**) tool.

The mSUPA tool maintained the non-parametric approach of univariate SUPA; linking tolerances to control limits. The novelty of this research is the non-parametric approach; multivariate SPC tools typically are parametric relying on the assumption that the process is multivariate normal. Even previous attempts to extend Pre-Control, a non-parametric tool, into the multivariate case have uncoupled control limits from tolerances and linked them to statistical limits based on a multivariate normal distribution.

A consideration that needs to be made in the implementation of mSUPA is its presentation. Although a product with multi-dimensional CtQ features can be theoretically plotted on a single chart, once a product has more than 2 CtQ features this becomes increasingly difficult to visualise. However, if the calculation was automated, the practitioner could be presented with a simple traffic-light describing if a process is in a global green, yellow or red zone. Then the simple SUPA rules can be applied; which leads to **RQ5**.

**RQ5** When a low-volume multivariate process control tool signals the process is off-target, how can the calculation of control parameter adjustments be automated?

This question is addressed by chapters 3 and 7 and raises a problem facing practitioners who are operating multivariate processes. Currently, how control parameter adjustments are made are left to a process operator's judgement. This creates variation between batches of products made by different operators. This research developed a novel tool that implements an online simulation that automates the calculation of control parameter adjustments. Using this tool in conjunction with mSUPA provides an overall control approach for discrete low-volume manufacture, that eliminates the rule-of-thumb methods commonly used.

## 8.3 Future Work

In this section, directions for future work based on the research in this thesis are outlined as follows:

1. Further work is required to identify the best means of presenting the information generated by mSUPA to aid the final implementation.

2. The online process adjustment simulation is underpinned by the assumption that the $A$ matrix that links the effect of a control parameter adjustment on the current process mean can be estimated. For example, if a product is manufactured in a machining centre, the product has an associated CNC program. It is hypothesised that an estimate of the $A$-matrix can be extracted from this program, hence, further work is needed in this area.

3. If the mSUPA and automated process adjustment tools are implemented, there is an opportunity for the software to directly adjust the process parameters. This reduction in operator influence has the potential to increase productivity.

# Chapter 9

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# Appendix A

# CtQ Outputs from Manufacturing Simulation

This appendix details experimental results taken to validate the PROVADT method. These results were collected from a manufacturing simulation known as PIM. Table A.1 details the results collected in the original PROVADT format and Table A.2 details the results collected in the enhanced PROVADT format. These results are analysed in detail in section 4.5.

Table A.1: Original PROVADT format outputs from manufacturing simulation

| Piece No. | Location 1 | Location 2 | Location 1 | Piece No. |
|---|---|---|---|---|
| | Appraiser 1 | Appraiser 1 | Appraiser 2 | |
| 1 | 20 | 30 | 20 | 1 |
| 2 | 58 | 87 | 54 | 2 |
| 3 | 36 | 45 | 35 | 3 |
| 4 | 76 | 52 | 78 | 4 |
| 5 | 17 | 47 | 18 | 5 |
| 6 | 43 | 68 | 49 | 6 |
| 7 | 58 | 94 | 55 | 7 |
| 8 | 77 | 96 | 78 | 8 |
| 9 | 22 | 56 | 24 | 9 |
| 10 | 43 | 21 | 43 | 10 |
| 11 | 60 | 77 | 58 | 11 |
| 12 | 78 | 34 | 77 | 12 |
| 13 | 22 | 72 | 22 | 13 |
| 14 | 47 | 75 | 44 | 14 |
| 15 | 43 | 17 | 44 | 15 |
| 16 | 90 | 63 | 90 | 16 |
| 17 | 20 | 33 | 23 | 17 |
| 18 | 35 | 65 | 38 | 18 |
| 19 | 47 | 63 | 46 | 19 |
| 20 | 70 | 45 | 74 | 20 |

Table A.2: Enhanced PROVADT format outputs from manufacturing simulation

| Piece No. | Location 1 | | | Location 2 | Location 3 | Piece No. |
|---|---|---|---|---|---|---|
| | Appraiser 1 | Appraiser 1 | Appraiser 2 | Appraiser 1 | Appraiser 1 | |
| 1 | 20 | 21 | 20 | 30 | 41 | 1 |
| 2 | 58 | 57 | 54 | | | 2 |
| 3 | 36 | 34 | 35 | 45 | 59 | 3 |
| 4 | 76 | 75 | 78 | | | 4 |
| 5 | 17 | 22 | 18 | 47 | 33 | 5 |
| 6 | 43 | 45 | 49 | | | 6 |
| 7 | 58 | 60 | 55 | 94 | 41 | 7 |
| 8 | 77 | 79 | 78 | | | 8 |
| 9 | 22 | 26 | 24 | 56 | 70 | 9 |
| 10 | 43 | 45 | 43 | | | 10 |
| 11 | 60 | 56 | 58 | 77 | 26 | 11 |
| 12 | 78 | 79 | 77 | | | 12 |
| 13 | 22 | 25 | 22 | 72 | 89 | 13 |
| 14 | 47 | 48 | 44 | | | 14 |
| 15 | 43 | 45 | 44 | 17 | 32 | 15 |
| 16 | 90 | 84 | 90 | | | 16 |
| 17 | 20 | 22 | 23 | 33 | 64 | 17 |
| 18 | 35 | 35 | 38 | | | 18 |
| 19 | 47 | 47 | 46 | 63 | 36 | 19 |
| 20 | 70 | 70 | 74 | | | 20 |

# Appendix B

# CtQ Outputs from Rettig Case Study

This appendix details experimental results taken from a case study at Rettig to validate the PROVADT method. These results were collected from two production lines, manufacturing radiators for a consumer market. The tables in this appendix show results for both production lines, RT1 and RT2, and for two features, radiator height and seam depth, on the samples collected. These results are analysed in detail in section 4.6.

Table B.1: Enhanced PROVADT CtQ outputs from Rettig case study for RT1 radiator height.

| Piece No. | Location 1 | | | Location 2 | Location 3 | Piece No. |
|---|---|---|---|---|---|---|
| | Appraiser 1 | Appraiser 1 | Appraiser 2 | Appraiser 1 | Appraiser 1 | |
| 1 | 448.06 | 448.14 | 448.16 | 447.91 | 448.04 | 1 |
| 2 | 447.79 | 447.84 | 447.89 | | | 2 |
| 3 | 447.98 | 447.93 | 448.25 | 448.01 | 448.02 | 3 |
| 4 | 448.01 | 448.05 | 448.06 | | | 4 |
| 5 | 448.25 | 448.3 | 448.32 | 447.79 | 448.56 | 5 |
| 6 | 448.43 | 448.42 | 448.6 | | | 6 |
| 7 | 448.21 | 448.39 | 448.25 | 448.43 | 448.58 | 7 |
| 8 | 448.18 | 448.2 | 448.37 | | | 8 |
| 9 | 447.63 | 447.7 | 447.68 | 448.18 | 448.18 | 9 |
| 10 | 448.17 | 448.2 | 448.18 | | | 10 |
| 11 | 447.66 | 447.72 | 447.6 | 448.21 | 448.25 | 11 |
| 12 | 448.19 | 448.16 | 448.24 | | | 12 |
| 13 | 447.64 | 447.58 | 448 | 448.12 | 448.73 | 13 |
| 14 | 448.16 | 448.14 | 448.38 | | | 14 |
| 15 | 448.09 | 448.12 | 448.3 | 448.17 | 448.44 | 15 |
| 16 | 448.27 | 448.49 | 448.39 | | | 16 |
| 17 | 448.43 | 448.39 | 448.65 | 448.44 | 448.62 | 17 |
| 18 | 448.35 | 448.55 | 448.36 | | | 18 |
| 19 | 448.49 | 448.48 | 448.42 | 448.31 | 448.54 | 19 |
| 20 | 448.34 | 448.5 | 448.79 | | | 20 |

Table B.2: Enhanced PROVADT CtQ outputs from Rettig case study for RT1 seam depth.

| Piece No. | Location 1 | | | Location 2 | Location 3 | Piece No. |
|---|---|---|---|---|---|---|
| | Appraiser 1 | Appraiser 1 | Appraiser 2 | Appraiser 1 | Appraiser 1 | |
| 1 | 10.96 | 11.54 | 10.88 | 10.78 | 10.92 | 1 |
| 2 | 11.3 | 11.44 | 10.64 | | | 2 |
| 3 | 11.24 | 11.58 | 11.2 | 11.7 | 10.94 | 3 |
| 4 | 11.5 | 11.26 | 11.28 | | | 4 |
| 5 | 11.02 | 11.22 | 11.04 | 10.44 | 10.98 | 5 |
| 6 | 11.96 | 12.04 | 11.72 | | | 6 |
| 7 | 10.8 | 11.06 | 10.52 | 11.4 | 10.62 | 7 |
| 8 | 11.96 | 11.2 | 11.32 | | | 8 |
| 9 | 10.54 | 10.2 | 10.48 | 11.3 | 10.98 | 9 |
| 10 | 11.18 | 11.56 | 11.24 | | | 10 |
| 11 | 10.54 | 10.4 | 10.46 | 10.08 | 10.96 | 11 |
| 12 | 10.94 | 11.22 | 11.26 | | | 12 |
| 13 | 10.94 | 10.92 | 11.18 | 10.7 | 11.16 | 13 |
| 14 | 11.52 | 11.48 | 11.54 | | | 14 |
| 15 | 11 | 11.08 | 10.98 | 10.58 | 11.04 | 15 |
| 16 | 11.7 | 11.64 | 11.68 | | | 16 |
| 17 | 12.32 | 12.22 | 12.46 | 12.42 | 12.22 | 17 |
| 18 | 11.7 | 11.08 | 12.14 | | | 18 |
| 19 | 11.9 | 11.82 | 11.92 | 12.34 | 11.84 | 19 |
| 20 | 11.04 | 11.22 | 11.28 | | | 20 |

Table B.3: Enhanced PROVADT CtQ outputs from Rettig case study for RT2 radiator height.

| Piece No. | Location 1 | | | Location 2 | Location 3 | Piece No. |
|---|---|---|---|---|---|---|
| | Appraiser 1 | Appraiser 1 | Appraiser 2 | Appraiser 1 | Appraiser 1 | |
| 1 | 526.45 | 526.39 | 526.24 | 526.75 | 526.99 | 1 |
| 2 | 526.28 | 526.11 | 526.09 | | | 2 |
| 3 | 526.37 | 526.25 | 526.79 | 527.04 | 527.22 | 3 |
| 4 | 526.18 | 526.25 | 526.2 | | | 4 |
| 5 | 526.66 | 526.64 | 525.55 | 526.12 | 526.64 | 5 |
| 6 | 526.59 | 526.85 | 526.42 | | | 6 |
| 7 | 526.24 | 526.34 | 525.67 | 526.7 | 526.83 | 7 |
| 8 | 526.61 | 526.57 | 526.54 | | | 8 |
| 9 | 526.33 | 526.68 | 526.54 | 526.8 | 526.6 | 9 |
| 10 | 526.16 | 526.18 | 526.45 | | | 10 |
| 11 | 526.13 | 526.24 | 526.21 | 526.9 | 527.06 | 11 |
| 12 | 526.66 | 526.96 | 527.07 | | | 12 |
| 13 | 525.73 | 525.55 | 526.12 | 526.69 | 526.83 | 13 |
| 14 | 526.77 | 526.74 | 526.75 | | | 14 |
| 15 | 527.2 | 526.83 | 526.84 | 526.8 | 527.06 | 15 |
| 16 | 526.76 | 526.86 | 526.72 | | | 16 |
| 17 | 526.61 | 526.72 | 526.42 | 527 | 526.69 | 17 |
| 18 | 526.66 | 526.43 | 526.69 | | | 18 |
| 19 | 526.03 | 526.61 | 526.22 | 526.26 | 526.19 | 19 |
| 20 | 526.22 | 526.32 | 526.57 | | | 20 |

Table B.4: Enhanced PROVADT CtQ outputs from Rettig case study for RT2 seam depth.

| Piece No. | Location 1 | | | Location 2 | Location 3 | Piece No. |
|---|---|---|---|---|---|---|
| | Appraiser 1 | Appraiser 1 | Appraiser 2 | Appraiser 1 | Appraiser 1 | |
| 1 | 10.34 | 10.6 | 10.94 | 10.96 | 10.76 | 1 |
| 2 | 10.68 | 10.42 | 10 | | | 2 |
| 3 | 10.34 | 10.36 | 10.16 | 10.78 | 10.64 | 3 |
| 4 | 9.7 | 9.82 | 9.36 | | | 4 |
| 5 | 9.66 | 10.2 | 9.54 | 10 | 9.54 | 5 |
| 6 | 10.52 | 10.46 | 10.64 | | | 6 |
| 7 | 9.44 | 9.24 | 9.58 | 9.92 | 9.56 | 7 |
| 8 | 9.96 | 9.7 | 10.68 | | | 8 |
| 9 | 10.22 | 9.78 | 10.3 | 10.02 | 10.22 | 9 |
| 10 | 9.38 | 9.2 | 9.34 | | | 10 |
| 11 | 10.38 | 10.72 | 9.42 | 9.78 | 9.92 | 11 |
| 12 | 10.28 | 9.94 | 9.66 | | | 12 |
| 13 | 8.78 | 8.7 | 8.78 | 9.84 | 9.76 | 13 |
| 14 | 10.02 | 9.24 | 9.72 | | | 14 |
| 15 | 10.32 | 10.3 | 10.14 | 10 | 10.18 | 15 |
| 16 | 9.84 | 9.5 | 9.82 | | | 16 |
| 17 | 9.46 | 9.88 | 9.32 | 10.34 | 9.62 | 17 |
| 18 | 10.36 | 10.02 | 10.36 | | | 18 |
| 19 | 9.54 | 10.02 | 10.36 | 10.58 | 10.24 | 19 |
| 20 | 10.38 | 10 | 9.92 | | | 20 |

# Appendix C

# CtQ Outputs from Coveris Study

This appendix details experimental results taken from a case study at Coveris to validate the PROVADT method. These results were collected from a manufacturing process, which extrudes plastic into sheet reels. The tables in this appendix show results for the material thickness on the samples collected. These results are analysed in detail in section 4.6.

Table C.1: Enhanced PROVADT CtQ outputs from Coveris case study for thickness.

| Piece No. | Location 1 | | | Location 2 | Location 3 | Piece No. |
|---|---|---|---|---|---|---|
| | Appraiser 1 | Appraiser 1 | Appraiser 2 | Appraiser 1 | Appraiser 1 | |
| 1 | 1.245 | 1.24 | 1.243 | 1.238 | 1.23 | 1 |
| 2 | 1.241 | 1.239 | 1.242 | | | 2 |
| 3 | 1.24 | 1.237 | 1.24 | 1.237 | 1.227 | 3 |
| 4 | 1.241 | 1.239 | 1.243 | | | 4 |
| 5 | 1.238 | 1.232 | 1.233 | 1.224 | 1.218 | 5 |
| 6 | 1.235 | 1.236 | 1.236 | | | 6 |
| 7 | 1.239 | 1.241 | 1.241 | 1.233 | 1.223 | 7 |
| 8 | 1.235 | 1.237 | 1.238 | | | 8 |
| 9 | 1.24 | 1.245 | 1.245 | 1.236 | 1.229 | 9 |
| 10 | 1.238 | 1.24 | 1.246 | | | 10 |
| 11 | 1.24 | 1.245 | 1.246 | 1.243 | 1.225 | 11 |
| 12 | 1.244 | 1.244 | 1.247 | | | 12 |
| 13 | 1.242 | 1.241 | 1.244 | 1.237 | 1.227 | 13 |
| 14 | 1.235 | 1.234 | 1.238 | | | 14 |
| 15 | 1.236 | 1.238 | 1.24 | 1.236 | 1.217 | 15 |
| 16 | 1.239 | 1.24 | 1.24 | | | 16 |
| 17 | 1.246 | 1.245 | 1.246 | 1.242 | 1.228 | 17 |
| 18 | 1.24 | 1.24 | 1.243 | | | 18 |
| 19 | 1.239 | 1.238 | 1.239 | 1.235 | 1.219 | 19 |
| 20 | 1.244 | 1.243 | 1.247 | | | 20 |

# Appendix D

# Case Study I Pre-Control Results

This appendix details experimental results taken from a case study to stage I PRE-Control as a process control method for set-up dominant processes. The tables in this appendix were plotted on percent tolerance PRE-Control charts to provide consistency. These results are discussed in detail in section 5.2.

| Part # | | | | | | Machine | | | | Date | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dimension ID | | | | | | | | | | | | | | | |
| Upper Tolerance | | | | | | | | | | | | | | | |
| Lower Tolerance | | | | | | | | | | | | | | | |
| Tolerance | | | | | | | | | | | | | | | |
| 0.5 Tolerance | | | | | | | | | | | | | | | |
| Target | | | | | | | | | | | | | | | |
| | Measurement | %Pre-Control | Action | Measurement | %Pre-Control | Action | Measurement | %Pre-Control | Action | Measurement | %Pre-Control | Action | Measurement | %Pre-Control | Action |
| 1 | | | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | | | | |

Figure D.1: Percent tolerance Pre-Control record example.


Figure D.2: Percent tolerance Pre-Control chart example.

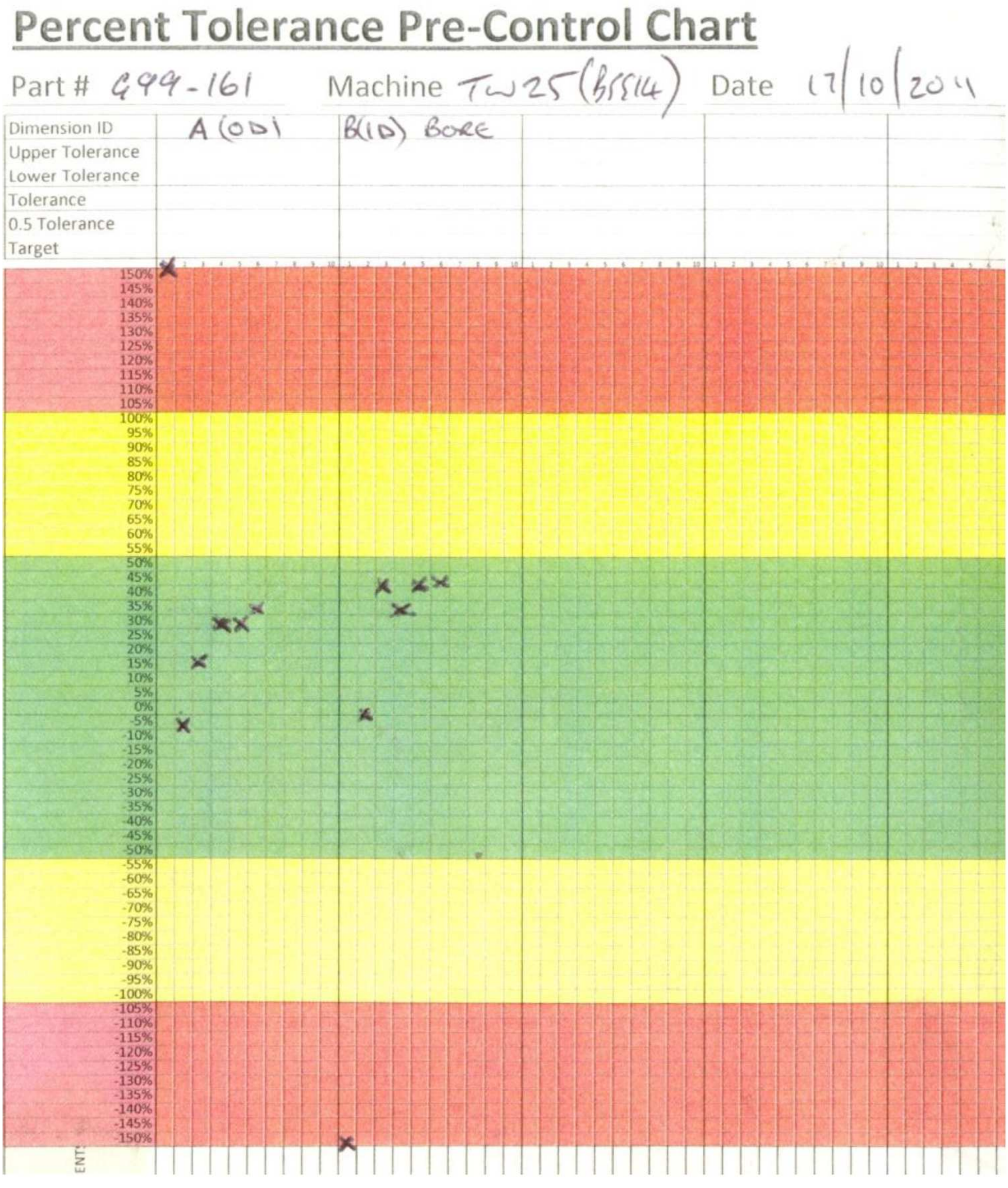


Figure D.3: Percent tolerance Pre-Control chart for G99-161.

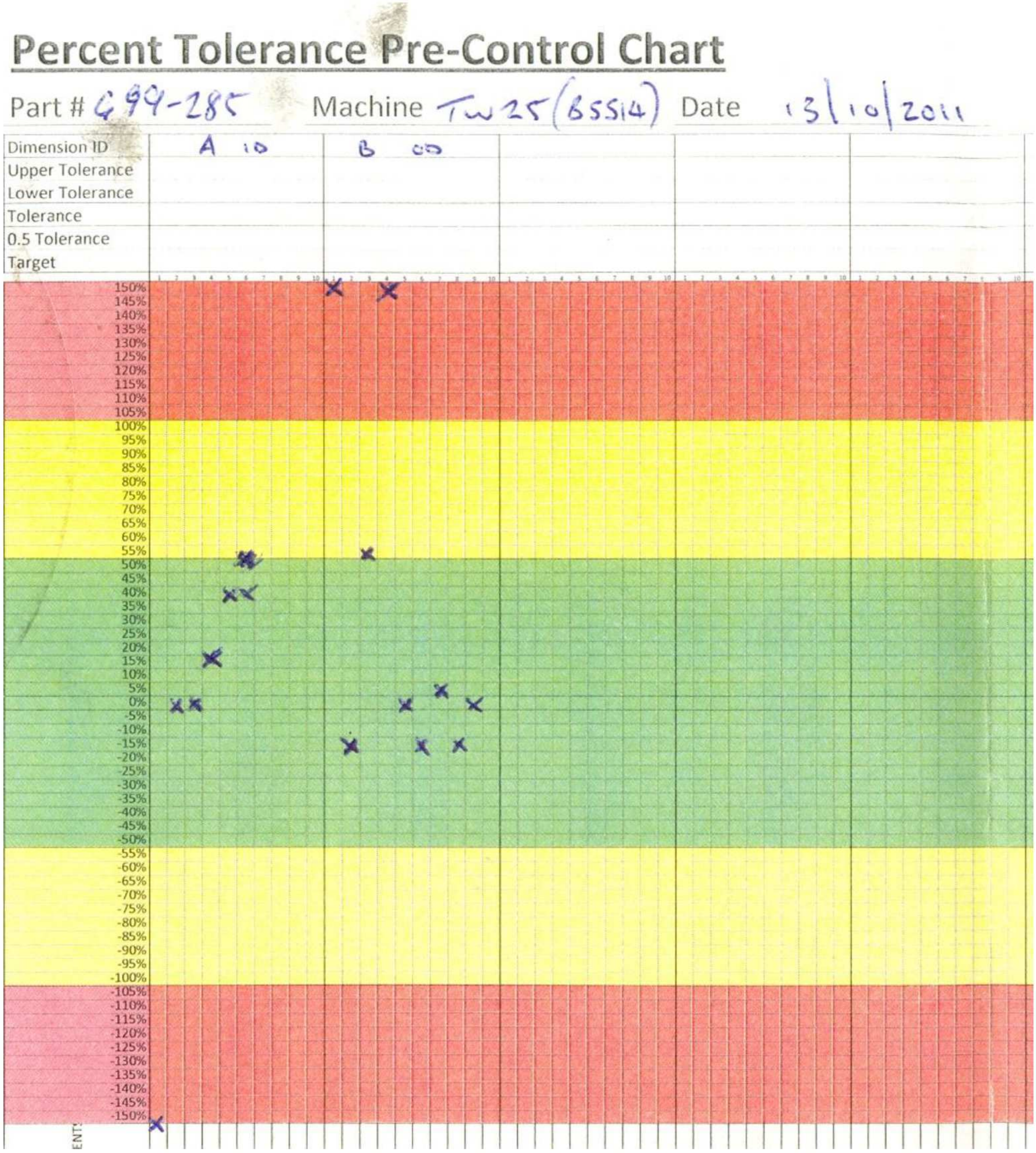


Figure D.4: Percent tolerance Pre-Control chart for G99-285.

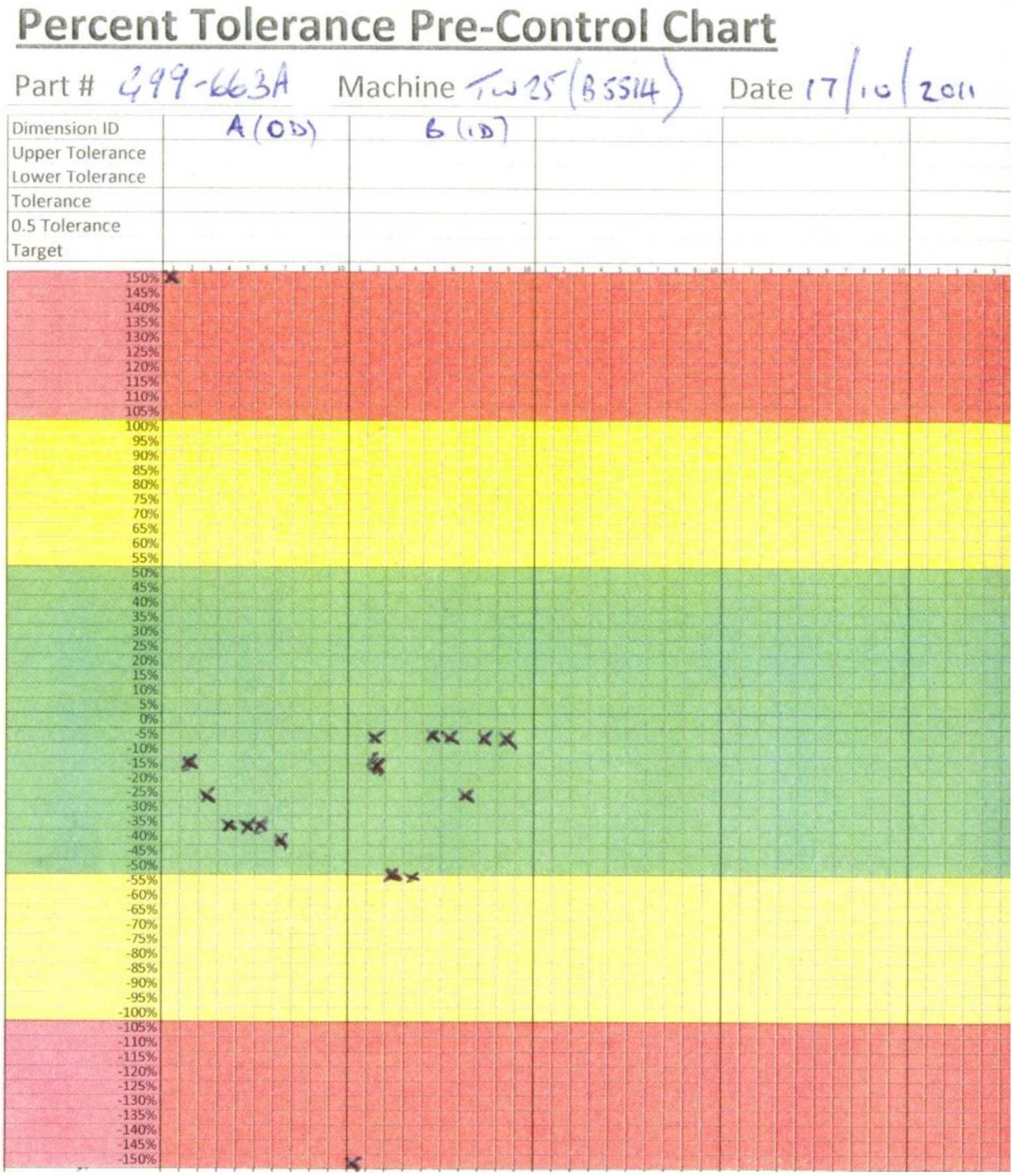


Figure D.5: Percent tolerance Pre-Control chart for G99-663A.


Figure D.6: Percent tolerance Pre-Control chart for W65-8487.


Figure D.7: Percent tolerance Pre-Control chart for G99-664A.


Figure D.8: Percent tolerance Pre-Control chart for W99-2780.